HYDROGEOLOGY
LECTURE
NOTES
EDITION 2.3

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# CHAPTER 14 – AQUEOUS GEOCHEMISTRY

**Introduction**

Aqueous geochemistry is the study of chemical processes in aqueous systems. These processes include reactions, mass transport, and the behavior of aqueous systems. Geochemical reactions are influenced by factors such as temperature, pressure, and the presence of other substances.

## Basic Chemistry Concepts and Definitions

- **Basic Chemistry Concepts and Definitions**: Essentials of chemistry that are fundamental to understanding geochemical processes.

## Atomic Mass and the Mole

Atomic mass and the mole are concepts fundamental to understanding chemical reactions and the stoichiometry of these reactions.

## Dispersion and Advection

Dispersion and advection are important processes in the transport of substances in groundwater.

## Potential Problems with Slug Tests

Potential problems with slug tests can arise from various factors, including the design of the test, the interpretation of results, and the environment in which the test is conducted.

## “Slug” Tests

“Slug” tests are a type of field experiment used to study the transport of substances in groundwater.

## Pumped Well Recovery

Pumped well recovery tests are used to determine the hydraulic conductivity of an aquifer.

## Boundary conditions

Boundary conditions are the initial and boundary conditions that define the problem in groundwater flow and transport.

## Recharge

Recharge is the process by which water is added to an aquifer from the surface.

## Local versus regional

Local versus regional flow systems refer to differences in the spatial scale at which flow processes are observed.

## Heterogeneity and anisotropy

Heterogeneity and anisotropy are important characteristics of flow systems that affect the flow of water and the transport of substances.

## Relationship between topography and flow systems

The relationship between topography and flow systems is critical in understanding how water moves through the subsurface.

---

### CHAPTER 13 – MASS TRANSPORT

**Introduction**

Mass transport in groundwater involves the movement of substances due to advection and dispersion.

## Advection

Advection is the transport of substances by the movement of the fluid itself.

## Dispersion

Dispersion is the spreading of substances due to the random movement of fluid particles.

## Reactions and Retardation

Reactions and retardation refer to the chemical processes that occur in groundwater and the effects of these processes on the transport of substances.

## Capture Zone Analysis

Capture zone analysis is a method used to determine the area of influence of a well or other discharge point.

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### CHAPTER 12 – FLOW MODELING

**Introduction**

Flow modeling is the use of mathematical models to simulate groundwater flow and transport processes.

## Models: A General Definition and Specific Details

Models can be categorized as qualitative, physical or analog, and mathematical.

## Why do we use process models?

Process models are used to predict the behavior of groundwater systems.

## Specific Types of Groundwater Models

- **Qualitative Models**: Models that describe processes qualitatively.
- **Physical or Analog Models**: Models that use physical or analog representations.
- **Mathematical Models**: Models that use mathematical equations to describe processes.

## The Method of Finite Differences

The method of finite differences is a numerical technique used to solve differential equations.

## Constructing Finite Difference Models: an Introduction to MODFLOW

MODFLOW is a widely used computer program for modeling groundwater flow.

---

### CHAPTER 11 – STUDYING FLOW SYSTEMS

**Introduction**

Studying flow systems involves understanding the characteristics of these systems and how they operate.

## How Do We Study Regional Flow Systems?

Studying regional flow systems involves using various techniques and data to understand the flow of water at large scales.

## What Data Can We Gather?

Data gathering is an essential part of studying flow systems.

## Surface Features

Surface features can provide important information about groundwater flow and transport.

## Hydraulic Heads

Hydraulic heads are used to determine the pressure at various points in a system.

## Water Balance

Water balance is the study of the quantities of water entering and leaving a system.

## Temperature Gradients

Temperature gradients can provide information about the flow of water.

## Water Balance

Water balance is the study of the quantities of water entering and leaving a system.

## Potential Problems with Slug Tests

Potential problems with slug tests can arise from various factors, including the design of the test, the interpretation of results, and the environment in which the test is conducted.

---

### CHAPTER 10 – FLOW SYSTEMS AND REGIONAL GROUNDWATER FLOW

**Introduction**

Flow systems and regional groundwater flow involve the study of large-scale groundwater systems.

## Characteristics of Flow Systems

- **Boundary conditions**: The conditions at the edges of a flow system.
- **Recharge**: The process by which water is added to an aquifer from the surface.
- **Discharge**: The process by which water is removed from an aquifer.
- **Local versus regional**: Differences in the spatial scale at which flow processes are observed.
- **Heterogeneity and anisotropy**: Important characteristics of flow systems that affect the flow of water and the transport of substances.
- **Relationship between topography and flow systems**: The relationship between topography and flow systems is critical in understanding how water moves through the subsurface.

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### CHAPTER 6 – STUDYING FLOW SYSTEMS

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## Hydraulic Heads

Hydraulic heads are used to determine the pressure at various points in a system.

## Surface Features

Surface features can provide important information about groundwater flow and transport.
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Chapter 1 – Introduction to Hydrogeology

The Properties of Water

Pure water (H₂O) is:
- Clear, colorless
- No discernable taste or smell
- At 1 atmosphere - Melting point: 0°C; Boiling point, 100°C
- Density at 4°C: 1.00 g/cc

Water is present throughout the solar system, but other than on Earth it generally exists as a gas or solid. Liquid water is essential for life as we know it, and it is one of the reasons why the Earth is such a unique place in the Solar System.

- Exceptions:
  - liquid water once existed on Mars and may still be in the subsurface
  - several moons of Jupiter (especially Europa) may have liquid water

There are a couple of things about water that make it so unique:

- Water is a polar molecule (Figure 1.1a)
- Water is highly cohesive (Figure 1.1b)

![Figure 1-1. Model of water molecules showing a) molecule polarity, and b) attraction between water molecules responsible for cohesiveness and surface tension (© Uliana, 2001)](image)

The polarity of the water molecules results in:

- High surface tension
- High solvent ability (water is the ‘universal solvent’)

Distribution of Water on the Earth

Over 75% of the Earth’s surface is covered by water. The vast majority of that is in the oceans and is unfit for human consumption. Most of the freshwater is locked up in the ice caps and in glaciers. Usable, available freshwater, in the lakes and rivers on the surface and in the underground reservoirs, is less than one percent of the total water in the world.

The distribution of the world’s water is presented in Table 1-1:

<table>
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<tr>
<th>Type of Water</th>
<th>Percentage</th>
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<tr>
<td>Salt water in oceans</td>
<td>97.2%</td>
</tr>
<tr>
<td>Ice caps &amp; glaciers</td>
<td>2.14%</td>
</tr>
<tr>
<td>Groundwater</td>
<td>0.61%</td>
</tr>
<tr>
<td>Soil moisture</td>
<td>0.005%</td>
</tr>
<tr>
<td>Atmosphere</td>
<td>0.001%</td>
</tr>
<tr>
<td>Fresh surface water</td>
<td>0.0009%</td>
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Table 1-1. The distribution of the world’s water
History of Hydrogeology

Early 17th century – groundwater was believed to be seawater pumped up into the land by either tidal forces or air pressure

Some major advances in the development of hydrogeology as a science:

- In the latter part of the 17th century Pierre Perra ult conducted hydrologic investigations in the Seine River basin
  o First person to use a water balance approach to hydrogeology; he established that the local annual precipitation was more than ample to account for the annual runoff.

- Henri Darcy, a French hydraulic engineer, concluded that the rate of flow of a fluid through a porous media is directly proportional to the energy loss and inversely proportional to the length of the path of flow (1856).
  o First quantitative characterization of groundwater flow

- Arsène Dupuit (another French engineer) extended Darcy's work and developed equations for underground flow toward a well, for the recharge of aquifers, and for the discharge of artesian wells.

- O.E. Meinzer (American scientist with the US Geol. Survey) published several seminal documents on the subject, including his 1923 USGS Water Supply Paper 489 (The occurrence of groundwater in the United States with a discussion of principals) and the 1942 book titled Hydrology.

- In 1935, C.V. Theis recognized the analogy between groundwater flow and heat flow. Why is this important?
  o At that time the mathematical characterization of heat flow was well developed, while the mathematics of groundwater flow were not
  o Understanding the mathematics of heat flow allowed him to develop analytical equations for flow to wells; these are widely used in aquifer pumping test analysis and drawdown predictions

- In 1940, M. King Hubbert developed the theory that describes flow in large groundwater basins
  o Identified the difference between regional and local flow systems

- Also in 1940, Jacob put forth groundwater flow theory that incorporates the elastic behavior of porous rocks

- In early 1960s, J. Tóth took Hubbert’s theoretical treatment of regional flow systems and Jacob’s mathematics and used a computer to generate one of the first computer groundwater models

- 1970’s and 80’s: environmental and contamination issues became important; also research on potential use of geothermal energy

- 1984: McDonald and Harbaugh publish the first version of MODFLOW

- 1990’s: drastic technological advances in personal computers makes groundwater modeling widespread and available to everyone

- Current issues and contemporary areas of research:
  o Groundwater resource studies, especially concerning water availability determination and long-term sustainability of groundwater resources.
  o Flow in “non-porous media” (i.e., fractures and karst conduits), especially related to flow modeling and contaminant transport
  o Effects of long-term climate change on groundwater resources
The Hydrologic Cycle

To understand the hydrologic cycle, we must first understand systems and the components of a cycle.

**Systems**

*Systems* are a way of conceptualizing real-world phenomenon. We can distinguish between three types of systems (Figure 1-2).

In an *open system*, mass and energy can move across the boundaries of the system. In a *closed system*, mass stays constant, energy can move across the boundaries. In an *isolated system*, neither energy nor matter can flux across the boundaries.

When we study and model natural systems, we can deal with them in two ways – as *lumped systems* or as *distributed systems*.

**Lumped systems** only deal with inputs and outputs as a function of time. These systems ignore the details of what’s going on within each reservoir or part of the system. When we deal with lumped systems, we are basically using some version of the *hydrologic equation* (also called the *continuity equation* or the *conservation of mass equation*):

\[ I(t) - O(t) = \frac{\Delta S}{\Delta t} \]  

(equation 1-1)

where:
- \( I(t) \): input to a system over time
- \( O(t) \): output from a system over time
- \( \Delta S/\Delta t \): change in storage with a change in time

**Distributed systems** deal with the particulars of how various parameters (e.g., temperature, pressure, salinity, etc.) vary throughout the system with respect to space as well as time. We will discuss distributed models in more detail later when we discuss modeling in general and groundwater flow models in particular.

**Cycles**

A *cycle* is defined as a dynamic system that contains the following four components:

1. A *substance*, *element*, or *set of elements* that are in flux (not necessarily a *chemical* element)
   - e.g., water
2. A set of *reservoirs* in which the element resides
   - e.g., the oceans, the ice caps
3. A set of *fluxes*, or processes that are moving the elements within reservoirs and from one reservoir to another
   - e.g., rivers, precipitation
4. Some source of *energy* that is driving the cycle

**Overview of the Hydrologic Cycle**

Water occurs throughout the Earth, from the outer reaches of the atmosphere to deep in the mantle (and possibly as deep as the core). Early in the history of the Earth’s formation, around the time that the Earth’s crust began to form, volcanic activity released lots of volatile gasses, including water, from the underlying mantle. These volatile gasses made up the early atmosphere and oceans. It is thought that almost all of the water that we find in the oceans, lakes, streams, atmosphere, and the subsurface today was outgassed at this time, and that this volume of water has been cycling around ever since.
The **hydrologic cycle** is defined as the set of reservoirs and fluxes which hold and move water through the atmosphere, on the surface, and in the subsurface of the Earth (Figure 1-3). With the exception of minor amounts of extraterrestrial water brought in by comets, and small amounts of water vapor that are lost to outer space at the upper reaches of the atmosphere, there is a constant volume of water in the entire water cycle. Within the cycle, there are various reservoirs holding water and various processes that move water within reservoirs and from one reservoir to the next.

Figure 1-3 shows the main reservoirs and fluxes, as well as the two energy sources that drive the cycle. Reservoirs in the water cycle include the oceans, atmosphere, rivers, freshwater lakes, the unsaturated soil moisture, the saturated groundwater, **connate** water in deep sedimentary rocks, **magmatic** water from the mantle, water in the ice caps and glaciers (the **cryosphere**), and water in plants and animals (the **biosphere**). The fluxes are all the processes that move water from one reservoir to the next (e.g., evaporation, infiltration) or within a reservoir (e.g., groundwater flow, ocean currents).

Figure 1-3. A generalized and simplified diagram of the hydrologic cycle (© Uliana, 2003, 2012)

**Thought Question:** what is the primary source of energy driving the hydrologic cycle?

*What is a secondary source of energy?*

In terms of the systems discussed in figure 1-2, we treat the hydrologic cycle like a **closed system** – the cycle has a set amount of water that does not change; however, energy does enter the system in the form of sunlight. The amount of water gained from (or lost to) outer space is so small that we can consider it negligible. Within the hydrologic cycle, we can identify and delineate a number of **open systems**. The oceans are an example – water enters the oceans from rain, surface runoff, and groundwater discharge, and water leaves the oceans through evaporation, through mineral reactions, and at subduction zones. A lake is another example of an open system.

The science of **hydrogeology** deals with a specific part of the hydrologic cycle – the part that is underground. The primary focus of this book is on the saturated **groundwater** flow systems near the surface of the Earth – i.e. within the first 1 or 2 kilometers below the surface. The water in the unsaturated zone (the moisture in the soil) and the flowing water on the surface of the Earth (the rivers and lakes) are also dealt with in separate chapters. Magmatic water in the mantle and lower lithosphere, **connate water** (water that was trapped in sediments when they were first deposited), and petroleum-related fluids are not dealt with in this book.
Groundwater

The strict technical definition of **groundwater** is any water that is found beneath the surface of the Earth. This definition includes

a) The moisture that is found in the pores between soil grains
b) The fresh to slightly saline water, found in saturated geologic units near the surface, which is used for drinking and irrigation
c) The extremely salty brines associated with petroleum deposits and deep sedimentary units
d) The water found in the lower lithosphere and in the mantle.

In this book, we are primarily concerned with b) – the fresh to slightly saline water found near the surface that is frequently used for domestic, agricultural, and industrial purposes. From this point on, when the book uses the term ‘groundwater’, it is referring to the water described above after letter b).

**Distribution of Water in the Subsurface**

Geologic materials (i.e., rock, soil, and sediment) always have some amount of empty space in them. This empty space is called the **pore space**, and the percentage of pore space by volume in a rock or sediment is called the **porosity**\(^1\). Within all naturally-occurring geologic units, the pore space always contains some amount of moisture.

Below the surface of the Earth, in the soil and upper layers of rock, the sediments and rock contain moisture; however, not all of the pore space is full (i.e. there is both air and moisture within the pore spaces). Just below the surface, the amount of moisture is small, and the amount of moisture tends to increase with depth. Eventually, the amount of moisture becomes so great that the pore space is completely filled with water and the soil or sediment is **saturated** (Figure 1-4).

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\(^1\) Several terms, like porosity, permeability, and aquifer, are introduced in this chapter and defined in a general way. Please be aware that these terms will be re-defined and discussed in much more detail in later chapters.

---

![Diagram showing the distribution of water in the shallow subsurface](© Uliana, 2001, 2012)
Therefore, in the subsurface, we can distinguish between two basic zones – the \textit{unsaturated zone}, and the \textit{saturated zone}. The boundary between the two zones is often called the \textit{water table}; later in the book it will be shown that the top of the saturated zone is not exactly at the same location as the water table.

**Recharge and Discharge**

As shown in figure 1-3, groundwater is a part of the dynamic hydrologic cycle, and water must somehow enter as well as leave the subsurface. Water entering the subsurface is called \textit{recharge}. Recharge to the subsurface is generally through \textit{infiltration} – percolation of surface water (from rain, perennial streams, melting snow, urinating dogs, etc.) downward into the soil. As the water percolates down through the soil, sediment, and rock, the percentage of the pore space that is filled with water (or the \textit{degree of saturation}) increases until it reaches 100\% (i.e. complete saturation).

Just as water enters the saturated zone, it must eventually leave it. Movement of water out of the saturated zone is called \textit{discharge}. Natural discharge can be through a spring, into the bed of a stream, lake or ocean, or via evaporation directly from the water table. Pumping of groundwater through \textit{wells} – holes drilled into the ground for the purpose of accessing subsurface fluids – is another way that water discharges from the saturated zone.

**Groundwater Flow**

Groundwater is always flowing, and the direction of flow is determined by the location of higher groundwater elevation. Note, however, that groundwater does not flow \textit{downhill}; rather, it flows from higher \textit{hydraulic heads} (or higher water elevation) to lower hydraulic heads. The distribution of hydraulic heads in the saturated zone determines the direction in which the water will flow.

The speed with which groundwater flows, also called the \textit{velocity} or \textit{flux}, is determined by the difference in hydraulic head and the \textit{permeability} of the sediment or rock through which it flows. Permeability is a number which describes the ease with which a fluid (like water) will move through a \textit{porous medium} (i.e. a rock, soil, or sediment which has enough pore space to allow water to move through it). Later in the book, it will be shown how the difference in head from one point to the next, and the permeability, can be used to calculate the velocity of the groundwater.

**Aquifers**

An \textit{aquifer} is a geologic unit that can store and transmit a sufficient amount of water to supply a well. The factors that determine if a geologic unit is an aquifer include the following:

1. The permeability must be high enough that flow can be maintained.
2. The aquifer dimensions must be great enough (i.e., there must be a significant saturated thickness) to supply water to a well
3. The quality of the water must be good enough for the intended use.

There are three basic types of aquifers – \textit{confined}, \textit{unconfined}, and \textit{perched} (Figure 1-5). These will be discussed in greater detail later in the book.

In the next chapter we will present the concept of hydraulic head, and start laying the groundwork for a quantitative understanding of fluid flow in the subsurface.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{aquifer_types}
\caption{Cross-section showing various types of aquifers (© Uliana, 2001, 2012)}
\end{figure}
Chapter 2 – Fluid Potential and Kinetic Energy

Introduction

One of the fundamental equations that govern groundwater flow is called Darcy’s Law (equation 2-1),

\[ Q = K A \frac{dh}{dl} \]  

(Equation 2-1)

where:

- \( Q \) = discharge \([L^3/T]\)
- \( A \) = cross sectional area \([L^2]\)
- \( K \) = hydraulic conductivity \([L/t]\)
- \( \frac{dh}{dl} \) = hydraulic gradient or change in hydraulic head (h) per change in distance (l) \([\cdot]\)

In plain English, this equation states:

*Discharge (i.e., volumetric flow) through a cross sectional area is directly and linearly proportional to the hydraulic gradient, and the constant of proportionality that relates discharge to the hydraulic gradient is a quantity called the hydraulic conductivity.*

We will get into the details of the equation later in the course, but for now, let’s focus on the little “h” in the hydraulic gradient term, i.e., *hydraulic head*.

Fluid Energy

In simple, everyday terms, we think of hydraulic head as an elevation. More specifically, hydraulic head is the elevation of water in a manometer in a pressurized water pipe (Figure 2-1), or in a piezometer (Figure 2-2).

- A manometer is a vertical tube in a pressurized water pipe used to measure pressure in the pipe.
- A piezometer is a vertical tube with an open or slotted interval (usually called the screened interval or just the screen) inserted into the ground and used to measure hydraulic head in an aquifer; it is basically a well constructed for the sole purpose of measuring groundwater levels.

In actuality, the water levels are a manifestation of the fluid energy in a groundwater system, and measuring a water elevation in a well is really a measurement of the energy present in the fluid at a certain point in a groundwater system (Figure 2-2).

An aquifer is a dynamic system of flowing water. The fluid energy varies throughout the system and is different from one point to the next. The result is that we find different water levels throughout the aquifer. These different water levels result in hydraulic gradients, and water movement is driven by those gradients.
What do we mean by “energy”?

Everything in the universe has some amount of energy associated with it, and that energy is present in various forms. Some sort of energy drives every natural process, and the key to understanding physical processes is in understanding the distribution of energy in a system.

**Potential Energy**: energy stored in a piece of matter or at a point in a system; generally associated with position or with thermodynamics of the system (elevation, pressure, chemical, thermal)

**Kinetic**: energy associated with motion (velocity)

At every point in an aquifer, the fluid possesses some total amount of energy that is the sum of all the potential and kinetic energies in the fluid.

As we previously stated, the fluid energy at a point in an aquifer manifests itself as the water level in a piezometer. So we could also say that the water level, or hydraulic head, represents the total energy in the aquifer at a given point, and we can use the various energy components of the hydraulic head (elevation, pressure, velocity, etc.) to understand the driving forces behind fluid motion in the subsurface.

**The Bernoulli Equation**

As previously stated, fluid energies (and, subsequently, water levels) vary from one point in an aquifer to the next. Let us recall the first and second Laws of Thermodynamics. The 1st Law of Thermodynamics states that energy is conserved in any system; i.e. in cannot be created or destroyed, and any changes in energy must be accounted for in any system (equation 2-2).

\[
\text{energy added} - \text{energy subtracted} = \text{change in total energy}
\]

This is basically the same as the conservation of mass (equation 1-1). We could also express the first law in terms of the difference in energy at two points in a dynamic system:

\[
\text{total energy}_{(\text{at point 1})} + \text{energy added/lost}_{(\text{between point 1 and 2})} = \text{total energy}_{(\text{at point 2})}
\]

(Equation 2-3)

The 2nd Law of Thermodynamics states that closed systems tend to move towards increasing entropy. In a dynamic system like an aquifer, water will move from a point of higher energy (i.e., lower entropy) to a point of lower energy (higher entropy); in other words, groundwater moves in the direction of decreasing hydraulic head. Please note that groundwater does NOT (necessarily) flow downhill – it flows in the direction of decreasing head.

The Bernoulli equation (equation 2-4) describes the total energy of a fluid at all positions along a flow path in a closed system and is basically an expression of the 1st and 2nd Laws of Thermodynamics.

\[
z_1 + \frac{p_1}{\rho_w g} + \frac{v_1^2}{2g} + H_a = z_2 + \frac{p_2}{\rho_w g} + \frac{v_2^2}{2g} + H_L + H_E
\]

(Equation 2-4)

where…

- \(z\) = elevation [L]
- \(p\) = pressure [M/L·t²]
- \(\rho_w\) = fluid density [M/L³]
- \(g\) = gravitational acceleration [L/t²]
- \(v\) = velocity [L/t]
- \(H_a\) = heat energy added [L]
- \(H_L\) = mechanical energy lost [L]
- \(H_E\) = heat energy extracted [L]

…and the subscripts represent two different positions along the flow path.

Let’s assume that we don’t add or subtract any heat energy from the system, and the only change is the loss of mechanical energy from one point to the next, we can rewrite the equation as:

\[
z_1 + \frac{p_1}{\rho_w g} + \frac{v_1^2}{2g} + I_1 = z_2 + \frac{p_2}{\rho_w g} + \frac{v_2^2}{2g} + I_2
\]

(Equation 2-5)
Where $I$ is the internal energy at each point (i.e., the rest of the potential energy not described by the other terms) and $I_2 - I_1$ is equal to $H_L$ in equation 2-4.

We can understand this equation by considering each individual term in the equation as representing a specific component of energy, i.e.:
- Elevation
- Pressure
- Kinetic (velocity)
- Other internal energies (thermal, chemical)

The equation therefore describes the change in energy from one point along a flow path to the next. The interesting thing about it to us as hydrogeologists is that, since it expresses all the components of energy in the same units (i.e., length), we can use it to compare the relative magnitude of the individual components. When we do that, we see that for most groundwater situations, we can ignore the internal and the kinetic (velocity) components because they are so much smaller than the elevation and pressure components. We can also use the equation to mathematically “convert” one form of energy to another (e.g., we can calculate how much a certain change in elevation will result in a change in pressure.)

**Hydraulic Head and Hydraulic Potential**

If we go with the assumption that we can ignore velocity and internal energy components when dealing with groundwater, we can drop all that out of the equation and express the fluid energy as the sum of the elevation and pressure components. That sum is what we call *hydraulic head*; in physical terms it is the fluid energy per unit weight, and in mathematical terms it is:

$$ h = z + \frac{p}{\rho_w}g $$

(Equation 2-6)

where:
- $h$ = hydraulic head [L]
- $z$ = elevation [L]
- $p$ = pressure [M/L·t²]
- $\rho_w$ = fluid density [M/L³]
- $g$ = gravitational acceleration [L/t²]

If we multiply both sides of the equation by the gravitational constant, $g$, we get a quantity called *hydraulic potential* ($\Phi$), which is the fluid energy per unit mass, or

$$ \Phi = g z + \frac{p}{\rho_w} $$

(Equation 2-7)

such that

$$ \Phi = gh $$

(Equation 2-8)

The hydraulic potential is simply a way of expressing the same fluid energy so that it is independent of gravity (in case you would want to compare an aquifer on earth with one on Mars or something like that). We will probably not use hydraulic potential in this book; hydraulic head is sufficient for the problems that will be dealt with here.

**Physical Description of the Components of Head**

From equation 2-6, we can see that hydraulic head is the sum of the elevation component (or *elevation head*) and the pressure component (or *pressure head*). The physical meaning of pressure head and elevation head are defined and described in Figures 2-3 a-b. These figures show and idealized cross section of a piezometer with a screened interval at the bottom. Figure 2-3a shows the depths and elevations measured in the field (relative to some datum, like mean sea level). Figure 2-3b shows the pressure, elevation, and total heads.
From 2-3b, we see that the elevation head is the height of the screened interval above the datum. Keep in mind that the datum is arbitrarily chosen – if we wanted, we could choose the bottom of the well as the datum, and the elevation head would be zero. However, it is important to realize that the head is really only important to us when we are looking at multiple wells in the same aquifer, and we need to have a constant datum for all those wells in order to make the comparison meaningful. Sea level is usually chosen, but it is not the necessary datum.

![Hypothetical well showing depths, elevations, and heads](image)

Figure 2-3. Hypothetical well showing depths, elevations, and heads (© Uliana, 2001, 2012).

We also see that the pressure head is the length of the column of water in the well above the screened interval. Keep in mind that pressure and pressure head are two different things; the pressure at the screened interval is the force per unit area of the column of fluid above that point, while the pressure head is the pressure divided by the product of the density of the fluid and gravitational acceleration and is manifest as the length of the column of water above the screen. If we know what the density of the fluid is and the length of the column of water in the well, we can calculate what the pressure is at the well screen.

Finally, we see that the total head is just the sum of the other two heads, or more generally, the height of the water level in the well above the datum, and brings us full circle to the concept of water levels reflecting fluid energy.

**Distribution of Heads (i.e., Fluid Energy) in a 3-D Aquifer**

A word about mathematical quantities and parameters: There are three basic kinds of quantities:

- **Scalar** have magnitude only (e.g., temperature)
- **Vector** have both magnitude and direction (e.g., velocity)
- **Tensor** have magnitude that varies with respect to direction (e.g., stress)

Head is a scalar quantity; it is measured at a point and it has a single magnitude that doesn’t vary with respect to direction. So, when we talk about heads, from a theoretical standpoint we are talking about the energy at an infinitesimal point in an aquifer. However, that aquifer exists in three dimensions, contains an infinite number of points, and the head varies from point to point.

The trends in that variation are what control the directions and magnitudes of flow. We call this variation the hydraulic gradient. The hydraulic gradient is defined as the change in hydraulic head over the change in length, and it is directly analogous to other physical gradients (topographic slope, thermal gradient, concentration gradients, etc.)

Since hydraulic heads exist in three dimensions, hydraulic gradients are not necessarily horizontal. In reality, most groundwater flow is generally in a horizontal direction, and it is often a realistic assumption to ignore fluid movement in a vertical dimension. However, there are many situations where vertical gradients (and, subsequently, vertical flow) are significant.
So What?
The important thing to realize here is that when we install a piezometer in an aquifer, and measure a water level in that piezometer, we are measuring the fluid energy at a point (in three dimensions) that is located at the well screen. Consider Figure 2-4. We see two wells completed to approximately the same depth, and a third well that is much shallower. The two deep wells indicate a gradient pointing to the right of the page; however, the water level in the shallow well (located between the other two) is much lower. This indicates that there is some sort of vertical gradient in the system. (It also could mean that the wells are completed into different intervals that are not connected, but that is a tale for another day…)

![Figure 2-4. Cross section showing vertical and horizontal gradients (© Uliana, 2012).](image)

This will come into play later in the book, when we start looking at the equations that govern flow in a three-dimensional porous media.
Chapter 3 – Porous Media

Introduction

Up to this point, we have discussed the nature of water and the distribution of fluid energy (i.e., hydraulic head) in a flow system. Now we will turn our attention to the material through which the water flows. This topic will deal with the various aspects and properties of porous media, including:

- Porosity
- Permeability and hydraulic conductivity of porous media
- Variability of these parameters with respect to location and direction
- Measurement of these parameters

Porosity

All geologic materials have some amount of pore space, or empty space, in them. The term porosity ($\phi$) refers to the fraction of the total volume of a rock or sediment that is pore space. More rigorously, it is defined as the volume of the voids divided by the total volume, or

$$\phi = \frac{V_{\text{voids}}}{V_{\text{total}}}$$  \hspace{1cm} (Equation 3-1)

where:

- $V_{\text{voids}}$ = volume of the voids [L$^3$]
- $V_{\text{total}}$ = total volume of the sample [L$^3$]

Porosity can also be expressed as a percentage (simply multiply the ratio by 100). Note that porosity is dimensionless. We can also identify a quantity called the Void Ratio ($e$), which is defined as the volume of voids divided by the volume of solids, or

$$e = \frac{V_{\text{voids}}}{V_{\text{solids}}}$$  \hspace{1cm} (Equation 3-2)

where:

- $V_{\text{solids}}$ = volume of the solids [L$^3$]

Geologic materials are never completely dry; there is always some volume of water in them. We can think about geologic materials as composed of multiple phases – a solid phase, a water phase, and a gas phase (Figure 3-1). In turn, the solid phase can be further divided into its individual mineral phases (Figure 3-2).

Each of the mineral phases in a rock or sediment sample has a volume and mass associated with it. Every fluid and every mineral has a density associated with it.
Density

Density has units of \([\text{M} \cdot \text{L}^{-3}]\) and is defined as the ratio of an object's mass to its volume. We can express it mathematically as:

\[
\text{density} = \frac{\text{mass}}{\text{volume}} \quad \text{(Equation 3-3)}
\]

Often, we use the symbol \(\rho\) (rho) for density. Table 3-1 lists densities of some common substances.

Density is not a constant; it varies with temperature (generally higher temp = lower density; lower temp = higher density) and pressure (higher pressure = higher density; lower pressure = lower density). Therefore, when density is reported, it is usually accompanied by a temperature and a pressure. If no temperature or pressure is given, it is usually implied that the density value is at ‘standard temperature and pressure’ (STP), which is 25°C and 1 atmosphere.

The density of water behaves differently than most other substances. As liquid water cools, the density increases as expected. However, when water hits about 4°C, it reaches maximum density (1.000 g/cm\(^3\)), then as it cools further, the density goes down. That is why ice floats on water (whereas the solid form of most other substances would be denser than the liquid form and would therefore sink.)

We can also express density as the unit weight of a substance, where unit weight is equal to the density of a substance times gravitational acceleration (9.8 m/s\(^2\)). Unit weight is simply a way of expressing density in terms of weights that we actually measure in the lab. Density can also be expressed in a non-dimensional way (i.e., without units) by taking the ratio of the density of the substance to the density of water. This non-dimensional ratio is called the specific gravity \((\gamma)\), and is expressed mathematically as:

\[
\gamma = \frac{\rho_{\text{substance}}}{\rho_{\text{water}}} \quad \text{(Equation 3-4)}
\]

Specific gravity allows us to record density data in a way that is independent of units. We can convert specific gravity values to density values in whatever units we want simply by multiplying by the density of water. Since the density of water is approximately 1 g/cm\(^3\), specific gravity is basically equivalent to density in g/cm\(^3\). Table 3-2 lists the specific gravities of some common rock forming minerals.

<table>
<thead>
<tr>
<th>Mineral Name</th>
<th>Specific gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>2.65</td>
</tr>
<tr>
<td>Calcite</td>
<td>2.71</td>
</tr>
<tr>
<td>Dolomite</td>
<td>2.85</td>
</tr>
<tr>
<td>Na-feldspar</td>
<td>2.62</td>
</tr>
<tr>
<td>K-feldspar</td>
<td>2.57</td>
</tr>
<tr>
<td>Muscovite mica</td>
<td>-2.80</td>
</tr>
<tr>
<td>Clay (kaolinite)</td>
<td>2.60</td>
</tr>
</tbody>
</table>

Table 3-2. Specific gravities of some common rock-forming minerals.

We can see from the table that the majority of the rock-forming minerals that we are likely to encounter have a specific gravity of about 2.65-2.7. Since most rocks that we find are made of minerals with specific gravities of about 2.7, we can usually use a specific gravity of 2.7 when estimating the density of rocks.

However, all rocks have some amount of void (i.e., empty) space in them, and a certain percentage of this void space contains water. When determining the density of a sample of rock or soil, we need to take into account the percentage of void space in the rock, and the percentage of that void space that contains water, in order to determine the bulk density of the sample. The bulk density is defined as the overall density of a sample of rock (including the void spaces, any water in the void spaces, and any other substances that might...
be incorporated into the rock, such as organic material), as opposed to the density of the individual minerals that make up the rock.

For example: if we consider a 1 cm$^3$ piece of solid quartz, we know that quartz has a specific gravity of 2.65 and a density of 2.65 g/cm$^3$, therefore the piece of solid quartz will have a mass of 2.65 grams. However, if we consider a 1 cm$^3$ piece of quartz sandstone, the quartz sandstone will contain a certain percentage volume of quartz grains, a percentage volume of empty space in between the quartz grains, and a percentage volume of water in the empty spaces. Since water has a specific gravity of 1.0 (at STP), and the mass of air is so small that it is negligible, the overall density (i.e., bulk density) of the sample of sandstone will be less than 2.65 g/cm$^3$, and the mass of the sample will be less than 2.65 grams. Or, if our sandstone contains a large percentage of a heavier mineral, like magnetite ($\gamma=5.2$) or galena ($\gamma=7.5$), the sample may have a density greater than 2.65 g/cm$^3$. When calculating the mass of a sample, we need to first determine the bulk density of the sample, and then use that instead of the density of the minerals that make up the rock to calculate mass.

In order to calculate a bulk density of a sample of rock or sediment, we need to know the volume percentage of each "phase" (the water in the void space is a phase, each mineral is a phase, etc.) and the density of each phase in the sample. Bulk density is calculated by multiplying the volume percentage of each phase by the density of that phase, then summing up the products. In mathematical terms:

$$\rho_{\text{bulk}} = \left( \rho_{\text{mineral-A}} \times \frac{\% \text{volume}_{\text{mineral-A}}}{100} \right) + \left( \rho_{\text{mineral-B}} \times \frac{\% \text{volume}_{\text{mineral-B}}}{100} \right) + \left( \rho_{\text{water}} \times \frac{\% \text{volume}_{\text{water}}}{100} \right)$$  (Equation 3-5)

As previously stated, there is usually some measurable quantity of water in the pore spaces of any rock or sediment sample. We have a number of ways of quantifying the moisture content of geologic materials.

**Moisture Content**

Moisture content can be expressed as **gravimetric** moisture content (i.e., moisture content by weight) or **volumetric** moisture content (i.e., moisture content by volume). The gravimetric moisture content ($\omega$) is the weight of water in the sample divided by the weight of the solids in the sample, or:

$$\omega = \frac{\text{weight}_{\text{water}}}{\text{weight}_{\text{solids}}}$$  (Equation 3-6)

Volumetric moisture content ($\theta$) is the volume of water in the sample divided by the total volume of the sample, or:

$$\theta = \frac{\text{volume}_{\text{water}}}{\text{volume}_{\text{total}}}$$  (Equation 3-7)

Note that volumetric moisture content is always some value less than the porosity of the rock. We can also express the moisture content as the degree of saturation ($S_d$), which is the percentage of the void volume that is filled with water, or:

$$S_d = \frac{\text{volume}_{\text{water}}}{\text{volume}_{\text{voids}}}$$  (Equation 3-8)

All of these quantities can be expressed as fractions or as percentages (multiply by 100).

If we combine the equations for porosity, moisture content, and degree of saturation, we see that they are all related such that the volumetric moisture content is equal to the porosity times the degree of saturation, or:

$$\theta = S_d \times \phi$$  (Equation 3-9)
Geologic Materials and Implications for Fluid Flow

Now that we have the basic concepts down, let’s discuss porosity in the context of real geologic materials.

**Primary versus Secondary**

We distinguish between two main types of porosity in geologic materials:

- **Primary Porosity**: void space that forms in the rock during deposition and diagenesis
  - e.g., empty space in between sand grains in a sandstone, vesicles in a basalt
  - generally greater storage but less flow

- **Secondary Porosity**: void space that forms post-diagenesis
  - fractures, dissolution features
  - generally less storage, but more flow

The idea of fluid storage vs. fluid flow will come into play later in the book where dual porosity systems (i.e., a fractured geologic unit that has significant primary porosity) are discussed.

**Effective Porosity**

We defined the porosity of a rock as the volume of voids over the total volume. However, in real rocks, not all of the porosity is available for fluid flow. Some voids are too small, or the pore throats are too narrow, or some may be isolated from the rest of the rock. The porosity available for fluid flow is called the **effective porosity** ($\phi_{\text{eff}}$).

Examples of materials with potentially high total, but low effective, porosity include:

- **clay** (lots of saturated void space, but fluids do not move)
- **vesicular basalts** (vesicles are not connected)

*Thought Question: Is porosity a scalar, vector, or tensor quantity? How about effective porosity?*

**Specific Yield**

If we saturate a rock and let it drain by gravity, a certain percentage of the water in the pore spaces will be held in by surface tension and narrow pore throats. The fraction that drains is the **specific yield** ($S_y$) and the fraction that is retained is called the **specific retention** ($S_r$). More rigorous definitions are:

$$S_y = \frac{\text{volume}_{\text{drained}}}{\text{volume}_{\text{total}}}$$  
(Equation 3-10)

$$S_r = \frac{\text{volume}_{\text{retained}}}{\text{volume}_{\text{total}}}$$  
(Equation 3-11)

Note that $S_y + S_r = \text{total porosity.}$

*Thought Question: what is the relationship between $S_y$ and $\phi_{\text{eff}}$?*
Grain Size and Packing

For an ideal situation where grains are perfectly round with uniform size and packing, porosity does not vary with grain size. Grain packing does affect porosity (Figures 3-4a and b), and grain sorting also affects porosity (Figures 3-4c and d).

Figure 3-4. a) cubic packing, b) rhombic packing, c) well-sorted = high porosity, d) poor sorting fills up the extra pore space and lowers the total porosity (© Uliana, 2001, 2012).

Permeability

We’ve talked about porosity, which is the empty space in a rock or sediment. This is an important parameter in hydrogeology; however, it is not the only quantity that we need to understand in order to characterize groundwater systems. Different materials will transmit fluids with varying degrees of difficulty. The ease with which a porous media transmits a fluid is called the permeability.

Darcy’s Law

In 1856, Henri Darcy experimented with fluid flow through a pipe filled with sand (Figure 3-5a). He forced water through the pipe under a hydraulic head (i.e., he filled up a reservoir at a higher elevation than the pipe and allowed gravity to drive water through the sand) and he measured:

- the rate of discharge through the pipe (Q)
- the loss in head (dh) across some length of the pipe (dl).

He concluded that discharge of water through the pipe is directly proportional to the energy loss and inversely proportional to the length of the path of flow (Figure 3-5b). Or, if we call the loss in head over the length the hydraulic gradient, he found that the discharge is directly proportional to the hydraulic gradient.

This statement is expressed mathematically as equation 3-12

\[
\frac{Q}{A} \propto \frac{\text{dh}}{\text{dl}}
\]  

(Equation 3-12)

What does this mean?

Notice that the data fall on a straight line when graphed, meaning that the proportionality is linear. Whenever you have a relationship where one quantity is linearly proportional to another, the slope of the line allows you to set the two quantities equal to each other. In general mathematical terms, this slope is a constant of proportionality or a coefficient of proportionality. In this specific equation, we call the constant the hydraulic conductivity (K). Inserting K and rearranging the equation gives us Darcy’s Law, or:
\[ Q = KA \frac{dh}{dl} \]  
\text{(Equation 3-13)}

If we divide through by the cross-sectional area, the volumetric discharge reduces down to a 1-dimensional discharge (i.e., velocity) called the \textit{darcian velocity} \((q)\), or:
\[ q = K \frac{dh}{dl} \]  
\text{(Equation 3-14)}

When we deal with real aquifers, we often use a quantity called the \textit{transmissivity} \((T)\), which is defined as the hydraulic conductivity multiplied by the saturated thickness of the aquifer \((b)\), or:
\[ T = Kb \]  
\text{(Equation 3-15)}

The importance of this value will become evident later in the book; for now just be aware of the definition of the term.

The hydraulic conductivity, therefore, is a quantity that characterizes the ease with which water moves through a porous media. It is important to realize that this relationship is for water specifically; if we are studying some other fluid (petroleum, high-salinity brine, molasses, etc) we need to use a different parameter.

**Permeability for Other Fluids**

Getting back to the basic physics: the ease with which a fluid moves through a porous media is dependent on:
- properties of the media itself (size of pores and connectivity) and
- properties of the fluid itself (density and viscosity)

The hydraulic conductivity, as measured and characterized by Darcy’s law, is for water only. This quantity not only characterized the properties of the media, but it also contains the properties of the fluid within it. Because of this, if you determine a hydraulic conductivity of a sand by putting it in a tube and running water through it, you cannot use that value to calculate the flow rate of, say, motor oil or mercury. This is not a big deal in most hydrogeology problems (since we generally deal with water in near-surface environments), but it does concern us if we are dealing with multiple phases (e.g., oil and water). To handle those problems, we use a quantity called the \textit{intrinsic permeability} \((k)\). This value is a quantity that characterizes the ease with which any fluid flows through a porous media, and only characterized the porous media itself. The equation for intrinsic permeability is:
\[ Q = k \frac{\rho_{\text{fluid}} g A}{\mu_{\text{fluid}}} \frac{dh}{dl} \]  
\text{(Equation 3-16)}

where:
- \(k\) = intrinsic permeability \([L^2]\)
- \(\rho_{\text{fluid}}\) = density of the fluid \([M/L^3]\)
- \(\mu_{\text{fluid}}\) = viscosity of the fluid \([M/L\cdot t]\)
- \(g\) = gravitational constant \([L/t^2]\)

This equation requires that the user specify the properties of the fluid (i.e., density and viscosity). Otherwise, it is basically the same as Darcy’s Law. The hydraulic conductivity and the intrinsic permeability are related in the following way:
\[ k = K \frac{\mu_{\text{fluid}}}{\rho_{\text{fluid}} g} \]  
\text{(Equation 3-17)}

**Other Famous Equations Analogous to Darcy’s Law**

The basic idea behind Darcy’s law is this:
- the discharge is driven by a gradient
discharge is linearly proportional to a gradient
- there is a constant of proportionality that characterizes the ease with which water flows through the system (a conductivity parameter)

In many physical sciences, we find other physical processes that, mathematically, are identical to groundwater flow in that they have the same three properties. Examples include:
- heat flow (Fourier’s Law)
- electricity (Ohm’s Law)
- diffusion (Fick’s Law)

Each one of these equations has a discharge rate, a gradient or potential, and a constant of proportionality that represents conductivity. The importance of this is that we can use solutions from these other sciences and apply them to groundwater flow problems. We will discuss a specific example of this in the chapter on Well Hydraulics (Chapter 7).

The Limits of Darcy’s Law

Although Darcy’s law is regarded as a fundamental relationship in the Earth sciences, it does have limitations, and there is a range of conditions over which it is valid.

Upper Limit

One of the assumptions inherent in Darcy’s law is the assumption of laminar flow. Darcy’s law is not valid above the onset of turbulence in the system, because the discharge is no longer linearly proportional to the gradient.

Lower Limit

There is a lower limit, called the threshold gradient, where fluid viscosity is too strong for the gradient to overcome the resistance to flow.

Darcy’s law is still widely used because 1) groundwater systems rarely become turbulent, and 2) at gradients near the threshold gradient, flow is generally so small we can ignore it.

Thought Question: in what geologic situations might we be concerned about the groundwater exceeding the limits of Darcy’s law?

Measuring Permeability

Understanding permeability is one thing; actually measuring it is something else. Unfortunately, no one has developed a simple permeability meter with a digital display that you can point at a rock or sediment and instantly measure the permeability. Instead, we have a variety of ways of measuring permeability, both in the field and in the laboratory. These methods generally involve forcing fluid through a sample of the material, measuring the head loss and discharge rate, and calculating permeability from the results.

- Permeameters or darcy tubes: basically a tube (like the one in Figure 3-5); fill it up with a sample, run water through, read the head loss and discharge, and calculate hydraulic conductivity
- Gas permeameters: use a gas (usually nitrogen) as the fluid instead of water. These can be portable for field use, or set up in a laboratory.
- Well/piezometer tests: basic idea is to either
  - Pump water out of a well at a constant rate, or
  - Add or remove a known volume of water from a well
  - You then monitor water levels, and use one of various methods to calculate permeability (we will get into this in much more detail later in the book).
- Empirical equations: people have noted correlations between permeability and properties of the rock, primarily grain size distributions, average grain size, or “effective” grain size. Some of these are given on the other handout.
- Inverse determinations: using field data (water levels, flow rates, dimensions of the aquifer) to ‘back out’ permeability values.

**Fluid Velocity**

In an earlier paragraph we stated that dividing the Darcy equation by the cross-sectional area reduces the volumetric discharge \( Q \) down to a one-dimensional discharge \( q \) that we called the specific discharge or darcian velocity. However, in reality, the water is not flowing through the whole cross-sectional area; it is only flowing through the pore spaces, and these make up only a percentage of the total area. Therefore, we have to modify the specific discharge by dividing the effective porosity to get a value called the average linear velocity \( v \), which is a more accurate characterization of the actual groundwater velocity. Or, in other words,

\[
v = \frac{q}{\phi_{\text{eff}}}
\]

(Equation 3-18)

However, the concept of groundwater velocity is even more complicated than that. The actual velocity of the groundwater varies from point to point (depending on the size of the pores, the tortuosity of the flow paths, and constriction at the pore throats). If we were able to measure flow velocities at a large number of points in an ideal aquifer, we would see a few that are really fast, a few that are really small, and a lot that are close to an average velocity. Plotting a frequency histogram would show us a normal distribution centered around the average linear velocity calculated above.

*Thought Question:* Why would we care about the variations in actual velocity when we can calculate the average linear velocity?

**Permeability Distributions in Aquifers**

Recall the definitions of scalar, vector, and tensor:

- Scalar: has magnitude only
- Vector: has magnitude and direction
- Tensor: has magnitude that varies with respect to direction

Permeability is a tensor quantity. Not only does it vary from place to place, but it also varies with respect to direction. We can use two vectors to graphically represent a permeability tensor and to describe the variations in permeability throughout a system.

When we talk about variations of permeability in an aquifer, we use two terms: heterogeneity and anisotropy.

- Heterogeneity: the parameter varies with respect to location
- Anisotropy: the parameter varies with respect to direction

Figure 3-6 graphically shows the relationship between these two terms. Be careful not to use these terms interchangeably – each refers to something specific.

![Figure 3-6. Graphical representation of the relationship between heterogeneity and anisotropy (© Uliana, 2001, 2012).](image)

**Statistical Distribution**

Permeability is a quantity that varies over 19 orders of magnitude! When we are looking at variations in an aquifer, and we measure a whole bunch of permeabilities, our measurements will usually cover several orders of magnitude. Therefore, we must be careful when averaging permeabilities to get a single value to characterize an aquifer.
Permeability is a log-normally distributed value (Figure 3-7). We are generally used to dealing with normal distributions, in which the values are centered on a mean value. However, a log-normal distribution means that the logarithm of permeability is distributed around a mean. This means that we can’t use the arithmetic mean to characterize our system; instead we must use a geometric mean (i.e., average the logs rather than the actual values.)

Specific definitions of the various types of means (i.e., averages):

- **Arithmetic mean**: add up the values and divide by the number of measurements
- **Geometric mean**: add up the logs of each value and divide by number of measurements
- **Harmonic mean**: divide the number of measurements by the reciprocal of the sum of the measurements

![Normal Distribution](image1)

![Log-Normal Distribution](image2)

Figure 3-7. Normal versus log-normal distributions (© Uliana, 2001, 2012).
Chapter 4 – Flow Equations

Introduction

Mathematics is a language that we use to express real-world phenomenon. The heart of this language is the equation. An equation is a way of expressing a quantity as a function of one or more other quantities.

Equations: A General Discussion

For someone not used to dealing with equations and mathematical statements, the key to understanding them is to think about the physical and conceptual relationships that the equation expresses. Let’s take a famous equation, and think about what it means conceptually.

\[ E = mc^2 \]  

(Equation 4-1)

Where \( E \) represents energy, \( m \) represents mass, and \( c^2 \) is a very large number, the square of the speed of light. What this equation means, in Einstein’s words, is:

It followed from the special theory of relativity that mass and energy are both but different manifestations of the same thing -- a somewhat unfamiliar conception for the average mind. Furthermore, the equation \( E \) is equal to \( mc^2 \), in which energy is put equal to mass, multiplied by the square of the velocity of light, showed that very small amounts of mass may be converted into a very large amount of energy and vice versa. The mass and energy were in fact equivalent, according to the formula mentioned before\(^2\).

The point of the equation is that matter (i.e., mass) and energy are fundamentally the same thing. What the equation tells us is that energy is equal to mass times a very large number (\([3.0 \times 10^8]^2\)\), indicating that a small amount of mass is equal to a large amount of energy. This is a very significant concept, especially to the residents of Hiroshima and Nagasaki, Japan, during the mid-1940’s.

This is how we need to approach equations and mathematics – by looking at the relationships between the various parts of the equation, and understanding how those parts relate to reality. To do that, try to follow this basic procedure:

1) look at all the variables in the equation and determine the significant variables vs. the constants
2) make sure that you have a basic understanding of what the variables represent (it helps to understand the dimensions, i.e., units, of the variables)
3) compare the variables on one side of the equation to those on the other and look for numerator-denominator relationships (numerator-denominator relationships means that quantities in the numerator are directly proportional, while quantities in the denominator are inversely proportional)
4) look at the other variables and quantities that are modifying the variables in question (exponents, logarithms, numerical constants…?)

What the hell does that mean? Let’s look at Einstein’s equation as an example (fill in the blank areas under each number).

1. The variables and constants are…

2. Energy is…
   (units?)
   mass is…
   (units?)

3. Energy and mass are related to each other how?

4. The constant (speed of light), and therefore mass, are modified how?
Put all this together and you see that energy and mass are equivalent (‘proportional’), and the constant of proportionality is the square of the speed of light. Since the square of the speed of light is such a large number, we can conclude that a small amount of mass is equivalent to a large amount of energy.

The advantage of the equation over a sentence that says basically the same thing is that the equation allows us to directly calculate how much energy we can get from a given mass of matter. This gives us the two things that make equations important to us:

1) they express real-world phenomena in a language that is widely understood and easy to communicate
2) they allow us to calculate and make predictions about the magnitude of one parameter based on valued of some other parameter.

Next, let’s try dealing with some equations that are a little closer to the subject matter of the course.

**Darcy’s Law**

As previously stated, Darcy’s law is

\[ Q = KA \frac{dh}{dl} \]  

(Equation 4-2)

Let’s go through the four steps and try to understand what the equation means:

1 and 2. The variables and constants are…

- **Q** = volumetric discharge – variable or constant?; units are [L³/t]
- **K** = hydraulic conductivity – variable or constant?; units are [L/t]
- **A** = cross-sectional area perpendicular to flow – variable or constant?; units are [L²]
- **dh** = change in head – variable or constant?; units are [L]
- **dl** = change in length along flow path – variable or constant?; units are [L]

3. How are the variables related? What are the numerator-denominator relationships

- between discharge (Q) and:
  - cross-sectional area
  - change in head (dh)
  - change in length (dl)
  - hydraulic gradient (dh/dl)
  - hydraulic conductivity

4. How are the variables modified? Are there any exponents or numerical constants?

Are the relationships linear, exponential, logarithmic…?

Using the above, express Darcy’s law in a sentence:

“The volumetric discharge is directly proportional to the cross-sectional area and the hydraulic gradient, and the constant of proportionality is a linear constant called the hydraulic conductivity.”

Let’s take another basic flow equation and give it the same treatment.

\[ Q = k \frac{\rho_{\text{fluid}} g A}{\mu_{\text{fluid}}} \frac{dh}{dl} \]  

(Equation 4-3)

where:

- **k** = intrinsic permeability [L²]
- **g** = gravitational constant [L/t²]
- **\rho_{\text{fluid}}** = density of the fluid [M/L³]
- **\mu_{\text{fluid}}** = viscosity of the fluid [M/L·t]

Do the same – 1) define the variables vs. the constants, 2) give the physical definition of the constant (with units), 3) determine any numerator-denominator relationships, and 4) determine the modifications of the variables.
Main Groundwater Flow Equations

Darcy’s law is the basic equation that governs the one-dimensional flow of water in a porous media, which states that groundwater discharge is proportional to the hydraulic gradient. So far, we have treated the gradient term as a simple finite mathematical quantity that can be calculated by dividing the drop in head from one point to the next by the distance between the two points (Figure 4-1).

![Figure 4-1. Cross section showing the overall gradient between two wells (© Uliana, 2001).](image)

It is not inappropriate to do this in practice; however, from a theoretical standpoint it may not entirely accurate. The water table could look something like this (Figure 4-2):

![Figure 4-2. Cross section showing the water table between two wells (© Uliana, 2001, 2012).](image)

In fact, the hydraulic gradient represents the slope of the water table or the potentiometric surface, and that slope varies from one point to the next. When we do hydrogeology in practice, we can find ways to ignore those variations and assume that they are not significant. However, you need to have a basic understanding of the theory and mathematics so that you can decide when it is appropriate to ignore those variations and when it is important to acknowledge them.

Differential Equations

Real, dynamic, three-dimensional systems (like aquifers) contain an infinite number of points, and the conditions in those systems vary from point to point. We use partial differential equations to describe and understand these systems.
A differential equation is an equation that contains a **derivative**. A derivative is simply the instantaneous change in a parameter with respect to some independent variable. In Darcy’s law, the hydraulic gradient is a derivative. (Figure 4.3)

![Figure 4-3. Illustration of hydraulic gradient as a slope along progressively smaller finite distances (Δh/Δl; figures a-c) and as an infinitesimal derivative at a single point (dh/dl, figure d) (© Uliana, 2005, 2012).](image)

In a differential equation, we use the symbol \( d \) to signify the infinitesimal change in magnitude of a parameter (e.g., head) with respect to some independent variable (e.g., length along the flow path). So, for Darcy’s law, we are looking at flow at a single point in a single direction (along the length of the flow path), and the derivative \( \frac{dh}{dl} \) represents the gradient in that direction at a single point.

Since groundwater flow (i.e., \( q \)) is driven by the gradient, \( q \) also varies from one point to the next. So, we can think about the change in flow with respect to length; or

\[
\frac{dq}{dl}
\]

But, as we previously acknowledged, an aquifer exists in three dimensions and contains an infinite number of points, and at each point groundwater is flowing in some direction. Let’s choose a single point in our aquifer, and consider how groundwater is flowing at that point (Figure 4-4).

![Figure 4-4. Vector resolved into component vectors in three dimensions (© Uliana, 2001).](image)

Groundwater at this point is flowing in some unique direction. We can represent the flow direction and magnitude with a vector (i.e., an arrow), and define a three-dimensional **coordinate system** (that’s the x, y, and z arrows; it gives us a frame of reference). Any vector within a coordinate system can be resolved into
component vectors in the principal (x, y, and z) directions. What this means is that each of the component vectors adds up to the actual flow vector.

Each one of those component vectors represents the flow (i.e., \( q \)) in each of the principal directions. There is a \( q \) in the x direction (\( q_x \)), in the y direction (\( q_y \)), and in the z direction (\( q_z \)). When we have a derivative that has multiple independent variables (i.e., the three directions), we use a partial differential equation, which uses the symbol \( \partial \) instead of \( d \) to indicate the change in magnitude of a variable with respect to the independent variable.

We can then express that flow vector by the following partial differential equation:

\[
\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = S \frac{\partial h}{\partial t} \tag{Equation 4-4}
\]

Now, remember Darcy’s law (expressed as the specific discharge) is:

\[
q = K \frac{\partial h}{\partial l} \tag{Equation 4-5}
\]

so, we can substitute in for \( q \), and we get the following equation.

\[
\frac{\partial (K_x \frac{\partial h}{\partial x})}{\partial x} + \frac{\partial (K_y \frac{\partial h}{\partial y})}{\partial y} + \frac{\partial (K_z \frac{\partial h}{\partial z})}{\partial z} = S \frac{\partial h}{\partial t} \tag{Equation 4-6}
\]

This is the primary groundwater flow equation, which describes the flow of groundwater at every point in a flow system.

**What does this mean in conceptual terms?**

This is just a more complex version of the continuity equation; each term on the left is...

\[I - O \text{ (what goes in minus what goes out)}\]

for each direction (x,y,z) for a given point in the flow system, and the term on the right is \( \Delta S \text{ (change in storage)} \)

for that point in the system.

In a 3D flow system, there are an infinite number of points, and an infinite number of equations that describe flow through each point. We cannot simply plug a few values into the equation and directly solve it; instead, we have to use some indirect method to solve the equation.
How Do We Solve These Equations?

Differential equations are not easy to solve directly, since there are technically an infinite number of equations within a system. We need to use one of several methods for solving the equations.

1) **Flow nets**
   - graphical representation of a groundwater flow system

2) **Analytical equations**
   - Solutions of the equation for specific situations
     - e.g., The Theis Equation – solves for radial flow to a well with a set of assumptions (horizontal flow, constant thickness, infinite extent…)

3) **Numerical Methods (Computer models)**
   - Reduce the infinite number of equations to a finite number (“discretization” and solve them with a computer
     - Finite difference (MODFLOW)
     - Finite element (FRAC3DVS, SUTRA)

These will be discussed in more detail in later chapters.
Chapter 5 – Storage Parameters
(or Flow Equations part 2)

Introduction

In the last chapter we discussed equations and introduced the main groundwater flow equation, which is:

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = S_s \frac{\partial h}{\partial t}$$  \hspace{1cm} (Equation 5-1)

where...

q = specific discharge (i.e., groundwater flow) in the x, y, or z direction [L/t]
h = hydraulic head [L]
t = time [t]
x, y, z = lengths in the principal directions in our coordinate systems

We also substituted Darcy’s law...

\[ q = K \frac{dh}{dl} \]  \hspace{1cm} (Equation 5-2)

where...

K = hydraulic conductivity [L/t]
\( \frac{dh}{dl} \) = hydraulic gradient, or change in head with length [-]

...for q, and we got the following equation.

\[ \frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} \right) = S_s \frac{\partial h}{\partial t} \]  \hspace{1cm} (Equation 5-3)

Now, the observant reader will notice that up to this point we have not defined the variable \( S_s \) on the right side, nor have we specifically discussed what the right side of the equation means. However, we did establish that this equation is basically the continuity equation, where the left side represents \( \text{I} - \text{O} \) (what goes in minus what goes out), and the right side represents \( \Delta S \) (change in storage).

\( S_s \) is a storage parameter called the specific storage, and it basically is a coefficient that relates the changes in head in the aquifer to the change in the amount of water stored in the aquifer at a certain point. In other words, it is a quantity that tells us how much water we can get out of a unit volume of aquifer for a unit drop in head. We will discuss specific storage, and the other storage parameters, in this chapter.

Aquifers

Before we discuss storage parameters, let’s review aquifers and the occurrence of groundwater in nature.

An aquifer is a geologic unit that can store and transmit a sufficient amount of water to supply a well. The factors that determine if a geologic unit is an aquifer include:

- permeability – must be high enough that flow can be maintained
- aquifer dimensions – there must be a significant saturated thickness to supply water to the well

If the aquifer unit is close to the Earth’s surface, and the sediments that comprise it are relatively permeable, the primary source of recharge to the aquifer will probably be infiltration of groundwater from precipitation that falls directly on the ground. In this case, a water table will form, and the aquifer is called a water-table aquifer or an unconfined aquifer (Figure 5-1).
- The water table is defined as the point where the pore-water pressure and the atmospheric pressure are equal.
- Water in a well will basically rise to the point where it was encountered during drilling.

\[ \text{RECHARGE} \]

\[ \text{WATER TABLE} \]

\[ \text{RIVER} \]

Figure 5-1. Unconfined aquifer (© Uliana, 2001, 2012).

If the regional geology consists of alternating layers of low- and high-permeability units (e.g., interbedded sand and clay), then the permeable layers that are overlain by the low-permeability units can form confined aquifers (Figure 5-2). Confined aquifers contain an additional component of pressure head that causes the water in a well to rise above the point where the well encounters the top of saturation. The low-permeability units are called confining layers.

\[ \text{RECHARGE} \]

\[ \text{POTENZIOMETRIC SURFACE} \]

\[ \text{WATER TABLE} \]

\[ \text{PERCHED AQUIFER} \]

\[ \text{UNCONFINED AQUIFER} \]

\[ \text{CONFINED AQUIFER} \]

Figure 5-2. Interbedded sand and clay units creating both confined and unconfined aquifers (© Uliana, 2001, 2012).

This additional component of pressure head is created by the difference in elevation between the recharge area and the point in the aquifer in which the well is completed.

This is basically the same thing we see in the Darcy tube – the reservoir to the left represents the recharge area, and the change in head from one manometer to the other represents the potentiometric (or piezometric) surface (Figure 5-3).
In addition to unconfined and confined aquifers, we can also have perched aquifers, which are saturated zones developed above isolated low-permeability units (like clay lenses) (Figure 5-2). These are basically unconfined aquifers that are not laterally continuous.

So… what we have established here is that there are two basic types of groundwater conditions. Unconfined conditions exist where the groundwater is not restricted by a confining layer and the pore fluid pressures at the top of saturation are equal to atmospheric pressure. Confined conditions exist where there is a confining layer and the water under the confining layer has an additional component of pressure that causes it to rise in a well to a point above the top of saturation. This point represents a point on the potentiometric (or piezometric) surface of the aquifer.

The reason we bring all of this up now is because the storage parameters that characterize an aquifer depend on the conditions we just outlined. So let’s move on to a discussion of storage parameters, focusing first on storage of water in confined aquifer.

Storage Parameters

Confined aquifers – a conceptual understanding

Let’s consider a confined aquifer that consists of a layer of sand overlain by a layer of clay that acts as a confining layer. If we stick a well into the aquifer and pump water out of it, we begin to lower the heads around the well in such a way that we create a cone of depression in the potentiometric surface (Figure 5-4). The more we pump from the well, the larger the cone of depression gets (both in terms of depth and areal extent). Now, if we look at the example in figure 5-4, we see that we have removed a volume of water (reflected by the volume in the barrel), and have lowered the water level in the well and in the surrounding aquifer. However, we have not changed the saturated thickness of the aquifer itself (i.e., the aquifer is still full up to the base of the confining layer). Where does the water come from? The conceptual answer deals with elastic deformation and compressibility of the aquifer materials and the water.

The aquifer is made up of sand grains with pore spaces that are full of water. If there is no confining layer and the aquifer is simply recharged by water infiltrating from above, the pore spaces are full of water and the pore fluid pressure is equal to atmospheric pressure plus the overlying column of water. In this case, you can fit a volume of water into the pore space that is equal to the volume of the pore space.
However, if the aquifer is confined and an additional component of pressure is added to the water (as shown in figure 5-4), the additional pressure will effectively ‘stuff’ a greater volume of water into the pores of the sand. It does this by compressing the sand grains slightly, pushing the sand grains around so that pore volume is increased, and compressing the water itself slightly. Increasing the pore fluid pressure causes the aquifer to undergo elastic deformation (i.e., the change in volume is recoverable), so that a subsequent decrease in fluid pressure will cause an associated decrease in deformation (Figure 5-5).

This is where the additional volume of water comes from. Pumping water from the confined aquifer lowers the excess fluid pressure (i.e., the pressure head above the top of the aquifer), causing the aquifer to ‘relax’ and yield water that is stored by the excess pressure.
Confined aquifers – a mathematical understanding

Now that we have a basic conceptual understanding of elastic storage of water in a confined aquifer, let’s treat this idea mathematically. What we as hydrogeologists are interested in knowing is what volume of water is associate with a given drop in head (or, conversely, how much head loss will be associated with a given withdrawal of volume).

Let’s consider a unit volume of our confined aquifer (by ‘unit volume’ I mean a cube with some convenient dimensions like 1 foot cubed or 1 meter cubed). This unit volume is saturated with water, and the water has an additional component of pressure that causes the water to rise above the top of the cube (Figure 5-6).

If we remove some volume of water from the cube, we will reduce the additional component of pressure and lower the head in the cube. Removing more water will cause more head loss; less water will cause less head loss. Therefore, we can find the volume of water that, if removed, will cause a unit drop in head (in this case, a head drop of one meter).

This number that we just measured (the volume of water per unit volume of aquifer that results in a unit drop in head) is the specific storage, given as $S_s$ in the groundwater flow equation.

The specific storage is an aquifer storage parameter that mathematically describes the volume of water that can be withdrawn from a unit volume of a confined aquifer for a unit drop in head. This value is also called the elastic storage coefficient, as it deals only with the amount of water removed from elastic storage. The specific storage is actually a function of the compressibility of the aquifer matrix (in this case, the sand) and the compressibility of the water; and is strictly defined as:

$$S_s = \rho_w g (\alpha + n\beta)$$

(Equation 5-4)

where:

- $\rho_w =$ density of water [M/L^3]
- $g =$ gravitational constant [L/t^2]
- $\alpha =$ compressibility of the aquifer matrix [1·M·L^{-1}·t^{-2}]
- $n =$ porosity [-]
- $\beta =$ compressibility of water [1·M·L^{-1}·t^{-2}]

Thought Question: What are the dimensions (i.e., units) of specific storage?

Because we are dealing with a unit volume of aquifer and a unit drop in head, the specific storage actually refers to a point in the aquifer. However, when we are dealing with real aquifers, the thickness of the aquifer has an influence on how much water the aquifer will yield; a thicker aquifer will transmit and yield more water than a thinner aquifer. If we multiply the hydraulic conductivity of the aquifer by the aquifer thickness, we get a quantity called the transmissivity (T), which describes the amount of water that can move through a unit width of the full saturated thickness of the aquifer under a hydraulic gradient of 1. Similarly, we have a
quantity called the **storativity** \( (S) \), which for a confined aquifer is the specific storage multiplied by the saturated thickness of the aquifer, or

\[
S = S_s b
\]

where:

\[
b = \text{saturated thickness of the aquifer} \ [\text{L}]
\]

In other words, the storativity describes the volume of water removed from a unit area of aquifer for a unit drop in head.

*Thought Question: What are the dimensions (i.e., units) of storativity?*

So far, we have only discussed confined aquifers. Next, we will discuss the response of unconfined aquifers to pumping, and talk about the storage parameters that are used to describe that response.

**Unconfined aquifers**

In a confined aquifer, all the water pumped out through a well is provided by the release of water in elastic storage. In an unconfined aquifer, we see the same thing – the changes in head result in a release of water from elastic storage in the aquifer. However, we are also removing water from the pore spaces and dewatering a section of the aquifer (Figure 5-7), and this aspect of the aquifer’s response adds an additional volume of water that is withdrawn.

This additional volume of water is released from the pore spaces and is due to the **specific yield** of the aquifer. Recall that specific yield is the volume of water that drains from a rock under the force of gravity, and is directly related to the effective porosity of the aquifer matrix. The volume of water released from a unit volume of aquifer due to dewatering of the aquifer is therefore just the unit volume times the specific yield.

So, in unconfined aquifers, the volume of water released from a unit volume of aquifer is a function of both the **specific yield** (i.e., the water removed from the pore space) and the **specific storage** (the water released from elastic storage in the aquifer). Therefore, in an unconfined aquifer, the **storativity** of the aquifer is the sum of the specific yield and the thickness times the specific storage, or:
\[ S = S_y + bS_s \]  \hspace{1cm} \text{(Equation 5-6)}

However, the magnitudes of these parameters are drastically different: specific yield (i.e., effective porosity) values in an aquifer generally range from 1% to 50%, or 0.01 to 0.5, while specific storage values generally range from \(1 \times 10^{-3}\) to \(1 \times 10^{-6}\), or 0.001 to 0.000001. This means that, in an unconfined aquifer, the volume of water provided by release from elastic storage is orders of magnitude less than that provided by dewatering the pores, so we can generally consider the specific storage as negligible and consider unconfined storativity as only a function of the specific yield.

Now, here is a thought question for class discussion:

Suppose you have two aquifers; each has the same porosity, permeability, and saturated thickness, but one is confined while the other is unconfined. If we place identical wells in each, and pump at the same rate for the same period of time, which will have the greater drawdown?
Chapter 6 – More on Flow Equations
(or Flow Equations part 3)

Introduction

Let’s review what we’ve talked about so far. We have presented the main equation that governs the three dimensional flow of water in a porous media, assuming that flow is within the limits of Darcy’s law (i.e., not so high that flow becomes turbulent, and not so low that it is below the gradient threshold). The equation is

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = S_s \frac{\partial h}{\partial t}$$

(Equation 6-1)

where...

- \( q = \) specific discharge (i.e., groundwater flow) in the \( x, y, \) or \( z \) direction \([L/t]\)
- \( x, y, z = \) lengths in the principal directions in our coordinate systems
- \( h = \) hydraulic head \([L]\)
- \( t = \) time \([t]\)
- \( S_s = \) specific storage \([1/L]\)

We also substituted Darcy’s law...

$$q = K \frac{dh}{dl}$$

(Equation 6-2)

where...

- \( K = \) hydraulic conductivity \([L/t]\)
- \( dh/dl = \) hydraulic gradient, or change in head with length \([-]\)

...for \( q \), and we got the following equation.

$$\frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} \right) = S_s \frac{\partial h}{\partial t}$$

(Equation 6-3)

This equation is basically the continuity equation, or

\( I - O \) (what goes in minus what goes out) = \( \Delta S \) (change in storage in the system).

In the chapter on flow equations we established that the left side of the equation describes the divergence of flux at any given point in the aquifer, which is a fancy way of saying \( I - O \) along each of the principal directions in our \( x, y, z \) coordinate system. Then, we put forth the idea that the right side of the equation describes changes in storage in the system, and we discussed the specific storage and other related storage parameter. As you recall, the storage parameters relate changes in head to changes in the volume of water stored in the aquifer. More specifically, the specific storage describes the volume of water that can be removed from (or added to) a unit volume of aquifer that will result in a unit drop (or rise) in hydraulic head.

The hydraulic head describes the total fluid energy at a specific point in an aquifer. This head changes from point to point, creating a tensor quantity called the hydraulic gradient, which reflects the Second Law of Thermodynamics (systems move from higher to lower energy; groundwater flows from high head to low head). Therefore, the hydraulic gradient is describing how head changes with respect to location in the aquifer. But heads in real aquifers do not just change from one point to the next; they can also change through time.

This brings us to the last part of the equation left to discuss: the partial derivative of head versus time, or \( \frac{\partial h}{\partial t} \), on the right side of the equation.
This term describes changes in head \((h)\) at a point in the aquifer with respect to time \((t)\), and combines with the specific storage to describe volume changes (i.e., changes in storage in the aquifer) with time. In this chapter, we will discuss time as it relates to groundwater, and then discuss some applications of these flow equations.

**Steady-state vs. Transient Groundwater Flow**

**Steady-state flow**

Consider the continuity equation; otherwise known as the law of mass conservation, or the continuity principal:

\[
I - O \ (\text{what goes in minus what goes out}) = \Delta S / \Delta T \ (\text{change in storage in the system}/\text{change in time}).
\]

This principal states that mass is conserved in a system, and any change of mass entering a system must be balanced by a corresponding change in mass in either the amount leaving the system or the amount stored in the system. In other words, if the amount going in equals the amount going out, then the amount in storage will not change, and \(\Delta S / \Delta T\) will equal zero. In the case where \(I\) is equal to \(O\), and \(\Delta S / \Delta T\) equals zero, we say that the system is at steady state. If our system is at steady state, heads at any given point in the aquifer do not change through time, the \(\partial h\) on the right side of the equation equals zero, and the equation becomes:

\[
\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = 0 \quad \text{(Equation 6-4)}
\]

or:

\[
\frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} \right) = 0 \quad \text{(Equation 6-5)}
\]

If we assume that the aquifer is homogeneous; that is, the permeability at one point is the same as at any other point, then the hydraulic conductivity \((K)\) will not change with respect to location, and we can remove \(K_x\), \(K_y\), and \(K_z\) from inside the differentials like so:

\[
K_x \frac{\partial}{\partial x} \left( \frac{\partial h}{\partial x} \right) + K_y \frac{\partial}{\partial y} \left( \frac{\partial h}{\partial y} \right) + K_z \frac{\partial}{\partial z} \left( \frac{\partial h}{\partial z} \right) = 0 \quad \text{(Equation 6-6)}
\]

If we also assume that the permeability of our aquifer is isotropic, then the hydraulic conductivity \((K)\) at any point in the aquifer will be the same in all directions, and \(K_x = K_y = K_z = K\). We can then factor out \(K\) and rewrite the equation as:

\[
K \left\{ \frac{\partial}{\partial x} \left( \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial h}{\partial z} \right) \right\} = 0 \quad \text{(Equation 6-7)}
\]

We can use a simpler notation for the derivatives and rewrite the equation as:

\[
K \left( \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} \right) = 0 \quad \text{(Equation 6-8)}
\]

Each one of the derivatives in the parentheses is a 2nd order partial differential. This just means that, with head, it isn’t just that head changes with distance, but the change in head with distance changes with distance.
(or, in other words, the gradient changes with distance). If we then divide both sides by $K$, the equation reduces to:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$  \hspace{1cm} \text{(Equation 6-9)}

a very famous equation called the Laplace equation. This equation describes the steady-state flow of groundwater (i.e., no changes in storage) in a homogeneous and isotropic aquifer. As we can see, the assumptions of homogeneity and isotropy greatly simplify the equation.

### Transient Flow

Steady state means that the amount going in equals the amount going out. In groundwater systems, this means that the amount recharging the system (infiltration of rain, loss from streams, leakage from other formations) is equal to the amount discharging the system (spring flow, discharge to streams, pumping by wells). In reality, recharge is never exactly equal to discharge (although, in some cases, they may be pretty close over the long term, and it is often appropriate to make this assumption when dealing with regional flow systems – more on that later). Real aquifers are in a constant state of dynamic change; recharge varies, discharge varies, the volume in storage varies, and head varies. If it rains more in one year than it usually does, recharge increases and heads will increase. If we pump a whole lot of water from a well, we will remove water from storage and heads will decrease. These scenarios depict transient flow conditions – conditions where heads and storage are changing through time.

The key point to consider here is time. It takes time for changes in discharge (e.g., pumping a well) to result in changes in head, and we as hydrogeologists are interested in knowing things like how much we can pump from our well in a day, how fast heads will drop when we pump, and how long it will take for heads to return to their original level if we stop pumping. That is what the right side of the equation refers to.

Remember, the specific storage is a coefficient, just like $K$ is a coefficient. A coefficient simply relates two quantities to each other:

- $K$ relates discharge to the hydraulic gradient. If we increase the gradient, we increase the discharge, and vice versa; $K$ tells us how much change in discharge we get for a given change in gradient.
- $S_s$ relates changes in head to changes in storage in the aquifer. If we remove a volume of water from storage, we lower the head in the aquifer, and vice versa; $S_s$ tells us how much change in head we get for a given change in storage.

The $\frac{\partial h}{\partial t}$ refers to the change in head with time, and the $S_s$ is the coefficient that states how much change in storage will occur with that change in head. So, this whole side of the equation really describes the change in storage in the aquifer with time, or:

$$\frac{\Delta S}{t} \approx S_s \frac{\partial h}{\partial t}$$  \hspace{1cm} \text{(Equation 6-10)}

So, to sum up:

- The three-dimensional groundwater flow equation is comprised of two parts: the divergence of groundwater flux in each of the three principal directions (the left side), and the change in storage in the aquifer with time (the right side).
- The right side is composed of two parts as well – the change in head with time, and a storage parameter that tells us the change in storage resulting from the change in head.
Flow Calculations and Applications of the Flow Equation

The following is a general overview of various ways of quantifying groundwater flow and solving, either directly or indirectly, the groundwater flow equation.

**Steady-state flow in a confined aquifer**

In the case of steady-state flow through a confined aquifer (Figure 6-1), the saturated thickness does not change.

![Figure 6-1. Steady flow through a confined aquifer of uniform thickness © Uliana, 2001, 2012.](image)

We can use a direct variation of Darcy’s law to calculate volumetric flow per unit width of aquifer ($q'$) like so:

$$q' = \frac{Kb}{dh} \frac{dh}{dl}$$  \hspace{1cm} (Equation 6-11)

where:

$q' = \text{flow per unit width of aquifer} \ [L^2/t]$

Notice that $Kb = T$, so we could substitute $T$ into the equation and use that.

The head ($h'$) at some intermediate distance ($x$) along the flow path can be calculated by equation 6-12:

$$h' = h_1 - \frac{q'}{Kb} x$$  \hspace{1cm} (Equation 6-12)

where:

$x = \text{some distance from } h_1 \ [L]$
Steady-state flow in an unconfined aquifer

Unconfined aquifers are a bit more complicated. The thing that makes them harder to deal with mathematically is that the water table is the upper boundary, and since the water table changes from one point to the next, the saturated thickness \( b \) changes as well (Figure 6-2). Since we can’t treat \( b \) as a constant, we have to use some integral calculus, and some specific assumptions, to develop an equation (based on Darcy’s law) that we can use directly.

We won’t go through the exact derivation here, but we will discuss the assumptions. This solution was first derived by Dupuit in 1863; hence they are called the Dupuit assumptions. This solution assumes:

1. the hydraulic gradient is equal to the slope of the water table
2. flow is horizontal (i.e., streamlines are horizontal, and equipotentials are vertical)

Incorporating these assumptions into the boundary conditions results in the Dupuit equation (equation 6-13):

\[
q' = \frac{1}{2} K \left( \frac{h_1^2 - h_2^2}{L} \right)
\]  

(Equation 6-13)

Calculating intermediate heads is a problem in an unconfined aquifer because unconfined aquifers are usually receiving recharge from above the water table (see Figure 5-1). Specific equations have been derived to calculate heads in an unconfined aquifer with areal recharge.
Flow lines and flow nets

Another way of solving steady-state groundwater flow problems is through the use of flow lines and equipotentials lines. A **flow line** is an imaginary line that traces the path that a particle of groundwater would follow as it flows through an aquifer. **Equipotential lines** are lines that connect points of equal head in the aquifer (the groundwater contours that you draw on a groundwater map are equipotentials). In an isotropic aquifer, the flow lines will be perpendicular to the equipotentials and will point in the direction of decreasing head. In an anisotropic aquifer, the flow lines will be deflected according to the direction of maximum K (Figure 6-3).

![Figure 6-3. Equipotentials and flow lines in isotropic and anisotropic aquifers (© Uliana, 2001, 2012).](image)

If the degree of anisotropy is known with certainty, there are ways that the amount of deflection (angle $\alpha$ in the anisotropic diagram, Figure 6-3) can be calculated.

A **flow net** is a graphical representation of the steady-state groundwater flow equation that depicts equipotentials and flow lines and, if properly drawn, represents a graphical solution to the 2 dimensional Laplace equation (i.e., horizontal, steady-state flow in a homogeneous and isotropic aquifer). Flow net construction requires the following assumptions:

1. the aquifer is homogeneous, isotropic, and fully saturated
2. the aquifer is at steady-state (heads are not changing with time)
3. flow is laminar and Darcy’s law is valid
4. all boundary conditions are known.

Let’s talk for a minute about boundary conditions. There are three basic boundary conditions that we need to understand when we are dealing with flow nets.

a) **no-flow boundary** – no gradient or flow across the boundary; the boundary represents a flow line examples?

b) **constant-head boundary** – no gradient or flow parallel to the boundary, boundary represents and equipotential line. examples?

c) **water table boundary** – the water table represents a boundary that is intersected by both flow lines and equipotential lines
Flow nets are two-dimensional, and can be drawn to represent either map-view or cross-sectional problems. Flow nets have to be drawn according to a specific set of rules; if so drawn they can be used to calculate the total discharge through the area. The key to drawing correct flow nets is to draw them in such a way that:

1. there is an equal change in head from one equipotential to the next (‘contour interval’)
2. there is a constant discharge between flow lines
3. flow lines cross equipotentials at right angles
4. the intersections of those lines create ‘curvilinear squares’, meaning that the area within the grids is basically square

The reason for this is because the area between the flow lines represents a flow tube, and the discharge in that flow tube is constant from one equipotential to the next. Therefore, according to Darcy’s law, if the gradient (the distance between consecutive equipotentials) increases, the discharge has to increase, and vice-versa. Now let’s think about this – if the gradient decreases (the equipotentials get further apart), discharge has to decrease (fewer flow lines in the same area, and each flow line gets further away from its neighbor). That’s what we see in the lower right corner of the flow net in figure 6-4 – as the equipotentials get further apart (gradient decreases), discharge per unit width also decreases, and the flow lines move further apart. Now, since gradient and discharge are linearly related, changing one will change the other by the same magnitude (e.g., double the gradient, you double the discharge), so the ‘grids’ will always be curvilinear squares.

When drawing a flow net, the first thing to do is to identify the boundary conditions and have them drawn in to scale. Usually, when doing a map-view flow net, you will be drawing the flow net directly onto the map, and that will already be taken care of.

Boundary conditions must be dealt with in the following ways (Figure 6-5):
The no-flow boundary represents a flow line – flow along the boundary is parallel to the boundary, and there is no gradient perpendicular to the boundary. When drawing a flow net with no-flow boundaries, that is the first flow line to draw in (because it is the only one of which you are sure).

Constant head boundaries, like lakes and streams, are points where the head does not change. For this reason, a constant-head boundary either represents a single equipotential (in the case of a lake or pond), or it represents a set of known elevation points (in the case of a stream or river). For lake/pond boundaries, the flow lines always intersect the boundary at right angles, and the nearest equipotentials should be close to parallel to the boundary. For streams, the equipotentials actually come into the stream at a tangent – at the exact point where they meet, the lines will be parallel. Streams usually complicate flow nets so much that, when constructing a rigorous flow net, they are really difficult to deal with. For simple groundwater contour maps, we just need to remember the rules concerning gaining vs. losing streams.

Water table boundaries are often used in constructing cross section flow nets. A classic cross-sectional flow net is the problem of flow through an earthen dam (Figure 6-6).

Once the flow net is drawn, the discharge through the flow net can be calculated by the following formula:

\[
q' = \frac{Kph}{f}
\]

(Equation 6-14)

where:
- \(q'\) = total volumetric discharge per unit width of aquifer [\(L^3/t\)]
- \(p\) = number of flow tubes (areas bounded by adjacent pairs of flow lines)
- \(h\) = total head loss over the length of the flow lines [L]
- \(f\) = number of equipotential drops
**Refraction of flow lines**

When groundwater moves from a formation with a specific hydraulic conductivity into a formation with a different conductivity, the discharge between each flow line stays the same. However, since $K$ is different, the gradient must change, and therefore the flow lines have to move either together or apart, depending on the change in $K$. Therefore, when flow lines cross into areas with different conductivities, they will refract or bend (similar to light moving through a substance with one optical velocity into one with a different optical velocity).

Knowing the conductivities of the two formations, we can calculate the angles of refraction using the following formula:

$$\frac{K_1}{K_2} = \frac{\tan \alpha_1}{\tan \alpha_2}$$  \hspace{1cm} (Equation 6-15)

with $K$ and $\alpha$ defined in figure 6-7.

The amount of refraction, and the direction in which the flow lines are refracted (relative to the line normal to the interface) is determined by the degree of difference and by which way the water is flowing (Figure 6-8)

**Figure 6-7.** Refraction of flow lines at the interface of two formations with different conductivities (© Uliana, 2001, 2012).

**Figure 6-8.** Refraction of flow lines a) low to high conductivity; b) high to low conductivity (© Uliana, 2001, 2012).

Low permeability formations tend to have vertical gradients and vertical flow, whereas higher permeability formations tend to have horizontal flow lines.
Chapter 7 – Well Hydraulics

Introduction

So far we have discussed the flow equations in an attempt to understand, both conceptually and mathematically, how groundwater flows in a porous media. The equation that we developed and discussed is a differential equation that governs flow at a given point under all conditions (assuming that Darcy’s law is valid); in essence, it is the all-purpose equation that covers all situations. As a differential equation, it cannot be solved directly unless we make some simplifying assumptions and apply it to a specific situation. One specific situation that has been solved for is the flow of water to a well. This is what we will discuss in this chapter.

Background Information on Wells

A well is a general, all-encompassing term for a hole drilled into the ground for the purpose of accessing fluids in the subsurface. Wells can be constructed to remove fluids (oil wells, water supply wells), to add fluids (waste and brine injection), or to sample and monitor the naturally-occurring fluids (monitoring wells). Wells have also been used to recover mineral resources in the subsurface; for example, subsurface sulfur deposits can be accessed by injecting extremely hot water, dissolving the sulfur, and pumping the concentrated sulfur solution up to the surface for recovery. Wells can also be used to access energy resources directly; for example, geothermal energy can be collected by injecting water into thermally active zones and recovering the heated water.

A well that is designed and installed for the sole purpose of measuring water levels in an aquifer is called a piezometer. Piezometers can be much smaller in diameter than water wells (generally 1.25 - 2 inch diameter). Piezometers can also be called monitoring wells. Monitoring wells are usually at least 2 inches in diameter, and are generally constructed for the purpose of periodic sampling of water for water quality (especially in contaminated areas that require monitoring through time).

Wells are an important aspect of applied hydrogeology, both from a practical standpoint (water supply wells, dewatering wells, etc.) in addition to a scientific standpoint (water levels, water sampling, etc.) Wells are the way we look into the subsurface and gather bits of information that we can use to understand the groundwater and the aquifer through which it flows.

Basic Well Construction

There are a number of different ways to drill a hole in the ground and install a well. Basic drilling methods include:

- **Cable tool** - uses a weighted drill bit that is repeatedly raised and dropped by a cable (similar to a pile driver). Seems archaic, but it is an effective method and results in minimal disturbance in the subsurface.
- **Water or mud rotary** – turn drill bit and circulate either water or drilling mud into the subsurface through the borehole. The fluid lubricates the bit and carries the drill cuttings up to the surface.
- **Air rotary** – similar to the fluid rotary methods, except air is used, and the bit (called a **down-hole hammer**) acts like a jackhammer run on compressed air. The air carries the cuttings to the subsurface.

Most water wells are drilled with rotary methods.

- **Auger** – rotate either a hollow-stem or solid-stem continuous flight auger without drilling fluids. Good for installation of shallow monitoring wells and piezometers.
- **Direct push** – uses the weight of the drill rig and hydraulic power to push the bit into the subsurface without rotation.
- **Jet drilling** – water is pumped under pressure down a small diameter pipe with a special bit; the pressurized water loosened sediments and rock that are carried out by the circulating water. The bit can also be lifted and dropped (like a cable tool rig) during drilling to help the bit advance.

Each one of these methods has advantages and disadvantages and is appropriate for certain applications.
Once the hole is drilled, the well must be installed. In certain situations, it may be acceptable to just leave the borehole open and start using it (for example, if you drilled a borehole in the middle of the Canadian Shield on exposed bedrock). However, in most cases, there is a possibility that the walls of the borehole may collapse in and block the well. Also, there may be several hydrostratigraphic units in one borehole; and it may be preferable to keep the upper ones isolated from the producing zone. Sections of pipe, called casing, are then placed in the well to isolate the upper units and keep the well open. Also, the producing zone may be unconsolidated or poorly consolidated sediments, or it may have some fraction of fine sediment associated with it, so some type of screen or filter pack must be placed in the zone to keep the mud out and let the water in. Figure 7-1 shows various geologic scenarios and different casing/screen setups.

![Figure 7-1. Examples of some typical well construction scenarios (© Uliana, 2001, 2012).](image)

Construction of monitoring wells is a little more rigorous than simple water wells. In a water supply well, you want to get as much water out as you can; therefore it is to your advantage to have a long screen length that covers a good part of the saturated thickness of the aquifer. In monitoring well, however, you want to be precise in what hydrostratigraphic unit you are monitoring. Several decades of shallow groundwater environmental assessments and remediation projects have generated relatively strict state and federally-mandated protocols concerning the installation of monitoring wells.

![Figure 7-2. Typical monitoring well construction (© Uliana, 2001, 2012).](image)

Figure 7-2 shows typical monitoring well construction. Monitoring wells and piezometers usually consist of 1½" or 2" diameter PVC pipe with threaded fittings and a cap at the bottom of the hole. The PVC pipe will have a section in the aquifer that has thin slots cut into it; this section of pipe is called the screened interval or slotted interval. (Note: screened interval is a general term that we use to describe the length of the well that is open to the aquifer). The open borehole around the screened interval is backfilled with clean, well sorted sand, called the sand pack or filter pack. The sand pack should fill in to a point at least a foot above the top of the screen. The sand pack is then covered with a seal, usually a smectitic clay called bentonite, which is purchased as dry pellets or a granular powder that swell when hydrated. This seals off the well screen from the upper part of the borehole and keeps surface water from moving down the borehole into the aquifer.
The remainder of the borehole is then backfilled with either Portland cement, drill cuttings, or some combination of the two. The well must then be covered with a manhole cover or some sort of secured covering.

Monitoring wells are usually installed using hollow-stem augers (Figure 7-3). Once the required depth is reached, the PVS pipe is put together and inserted down the center of the auger. Sand is then poured in around the pipe, and the augers are pulled up a little bit so that the sand falls into the open borehole. This process is repeated until the screen is covered and the sand is in place. Then the bentonite pellets are added, and the augers are pulled out of the ground. Water is poured in to hydrate the bentonite, and the rest of the hole is backfilled or cemented in.

**Figure 7-3. Hollow stem augers (from Lapham, et al, 1996).**

### Well Development

One of the big concerns with well construction is the amount of disturbance to the subsurface caused by drilling. Regardless of the method, the aquifer formation is always affected in some way. Specific disturbances:

- Lots of fine material gets into well screen and filter pack; can clog or silt up well
- Drilling fluids (especially muds) will get into the formation and clog pores
- Drill bit can smear the walls of the borehole, closing off pores.
- Excessive water must be removed

Following drilling and installation of the well, the effects of this disturbance must be corrected as best as possible. This process is called development of the well. Well development in water supply wells with relatively high yields generally entails pumping water from the well for an extended period of time, until the water runs clear. In low-yield monitoring wells, if there is not enough water to sustain pumping then it may not be an option. Plungers can be put into wells to force water in and out of the screen for a period of time. Or compressed air can be forced down into the well to push fine material out of the well screen. Usually, with monitoring wells, any water that was introduced during drilling must be removed so that any groundwater samples taken actually represent the formation.

Well Hydraulics

Now that we’ve discussed wells themselves, let’s talk a little about how groundwater flows to them. To understand the hydraulics behind flow to wells, we need to simplify the system and start with some basic assumptions:

1. the aquifer is homogeneous and isotropic
2. the aquifer is bounded on the bottom and top by confining layers
3. all geologic formations are horizontal and have infinite horizontal extent (we will deal with variations on this assumption later)
4. the potentiometric surface of the aquifer is horizontal prior to the start of pumping, and water levels are at steady state
5. any changes to the position of the potentiometric surface are due to the effect of the pumping well alone
6. all flow in the aquifer is horizontal
7. all flow to the well is radial
8. Darcy’s law is valid
9. groundwater has a constant density and viscosity
10. all wells are fully penetrating
11. the diameter of the pumping well is negligible

Be aware that, as we go on with the discussion of well hydraulics, some of these assumptions will be modified and others may be added to specific situations.

Horizontal Radial Flow

We have discussed and developed the equation that governs the flow of groundwater in three dimensions. That equation is:

\[
\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = S_s \frac{\partial h}{\partial t}
\]

(Equation 7-1)

where...

\(q = \) specific discharge (i.e., groundwater flow) in the \(x, y,\) or \(z\) direction [L/t]
\(x, y, z = \) lengths in the principal directions in our coordinate systems
\(h = \) hydraulic head [L]
\(t = \) time [t]
\(S_s = \) specific storage [1/L]

We also substituted Darcy’s law...

\[q = K \frac{dh}{dl}\]

(Equation 7-2)

where...

\(K = \) hydraulic conductivity [L/t]
\(dh/dl = \) hydraulic gradient, or change in head with length [L/t]
…for \( q \), and we got the following equation.

\[
\frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} \right) = S_z \frac{\partial h}{\partial t}
\]  
(Equation 7-3)

Now if we assume that our aquifer is homogeneous and isotropic (see ‘Chapter 06 – More on Flow Equations’), but not necessarily steady-state, the equation becomes

\[
K \left( \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} \right) = S_z \frac{\partial h}{\partial t}
\]  
(Equation 7-4)

Assumption \#6 states that all flow is horizontal. This means that there is no vertical component of flow, and therefore, the ‘z’ term drops out of the equation and it becomes

\[
K \left( \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) = S_z \frac{\partial h}{\partial t}
\]  
(Equation 7-5)

Now, recall that transmissivity (\( T \)) is defined as \( T = K b \) (where \( b \) = saturated thickness of the aquifer) and storativity (\( S \)) is defined as \( S = S_z b \). If we multiply both sides by \( b \), we get

\[
K b \left( \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) = S_z b \frac{\partial h}{\partial t}
\]  
(Equation 7-6)

or

\[
\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S_z}{T} \frac{\partial h}{\partial t}
\]  
(Equation 7-7)

If our aquifer is receiving recharge, either from infiltration of rain in an unconfined aquifer, or from leakage through the confining layer in a confined aquifer, we can add that directly to the equation and get

\[
\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{w}{T} = \frac{S_z}{T} \frac{\partial h}{\partial t}
\]  
(Equation 7-8)

where \( w = \) rate of vertical leakage or recharge \([L \cdot t^{-1}]\)

Now the thing about flow to a well is that it is radial; in other words, flow lines are converging on the well from all sides (Figure 7-4).

Because the flow lines converge, the equipotential lines that define the cone of depression that forms around the well will get closer together.

Figure 7-4. Radial flow to a well (© Uliana, 2001).
Now, if we are dealing with radial flow to the well, we need to express the equations in terms of a **polar coordinate system** (rather than the simple $x, y$ Cartesian coordinate system). In polar coordinates, a point is defined by its location with respect to a fixed central pole (in this case, the well). The location of a point is defined by its radial distance from the pole ($r$) and the angle between the radial line and some fixed polar axis (for example, north). Polar coordinates are graphically described in figure 7-5.

We can use the Pythagorean Theorem and a little basic geometry to re-state our two-dimensional flow equations in polar coordinates; when we do so, we get

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t}$$

(Equation 7-9)

and

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} + \frac{w}{T} = \frac{S}{T} \frac{\partial h}{\partial t}$$

(Equation 7-10)

So... what we have here is a differential equation that describes radial horizontal flow to a well based on a polar coordinate system. But just like before, this is a complex equation that requires some crazy math stuff (like Fourier transforms, Bessel functions, and Laplace transforms). Fortunately for us, these solutions have already been done, and all we have to do is get a little understanding of what they mean and how they are used.

### The Theis Equation

Earlier in the book we discussed the fact that the mathematics that govern the flow of groundwater (i.e., groundwater discharge is proportional to a gradient) are identical to those that govern a lot of other physical processes (like the flow of heat in a solid, the diffusion of solutes in solution, and electrical current). C.V. Theis noted the similarity between groundwater flow and heat flow, and he noted that pumping a well was mathematically analogous to heating a block of metal with a rod (Figure 7-6).

This situation assumes that the block has a constant thickness and an infinite extent (even though it’s drawn here as a block with finite proportions). If we start with the block at a constant temperature, then start heating the rod with a constant temperature, heat will be transferred to the block. It will take time for heat to move through the block (depending on the thermal conductivity of the metal).
The part of the block immediately adjacent to the rod will heat up first, then heat will flow through the block and, over time, develop a distribution of temperatures like that in figure 7-6. Conversely, if we start with a warm block of metal and cool the end of the rod, heat will also flow out of the metal into the rod and set up a similar distribution of temperatures. This is exactly the same thing that happens in an aquifer pumped by a well, except that water is flowing instead of heat, and a distribution of heads form instead of temperatures. That distribution of heads in an aquifer is the cone of depression.

The equation that Theis developed (Theis, 1935) is as follows (it looks really complicated at first, but the final equation is pretty straightforward)

\[ s = \frac{Q}{4\pi T} W(u) \]  
(Equation 7-11)

where

\[ u = \frac{r^2 S}{4 T t} \]  
(Equation 7-12)

and \( W(u) \) is the well function, which is the integral of \( u \) from \( u \) to infinity (equation 7-13):

\[ W(u) = \int_{u}^{\infty} \frac{e^{-u} du}{u} \]  
(Equation 7-13)

which can also be expressed as an infinite series that looks something like equation 7-14.

\[ W(u) = -0.5772 - \ln u + \frac{u^2}{2!} + \frac{u^3}{3!} - \frac{u^4}{4!} + \frac{u^5}{5!} - \text{yada yada yada...} \]  
(Equation 7-14)

The variables are defined as follows:

- \( s = \) drawdown [L], defined as the initial head minus the head at time \( t \)
- \( Q = \) pumping rate from the well [L^3/t]; assumed to be constant
- \( T = \) aquifer transmissivity (where \( T = Kb \)) [L^2/t]
- \( r = \) radius (distance from pumping well) [L]
- \( S = \) storativity (where \( S = S_h \)) [-]
- \( t = \) time after the start of pumping [t]

An important thing to note at this point is that the situations we are describing are transient; in other words, we are interested in understanding how heads in an aquifer change with respect to time when water is pumped from a well at a constant rate. That is what the Theis equation calculates – the drawdown at a point in the aquifer (some distance from the pumping well, \( r \)) at some time (\( t \)) after the beginning of pumping in response to a well pumping at a constant rate. If you know the transmissivity and storativity of a confined aquifer, then this equation can therefore be used to predict how much drawdown a pumping well will generate in that aquifer at some distance from the pumping well and at some time after the start of pumping.
To use the Theis equation, you need to have a table of values of $W(u)$ for values of $u$ (table 7-1).

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<td>32.02</td>
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Table 7-1. Values of $W(u)$ for values of $u$ (from Wenzel, 1942)
Chapter 8 – Aquifer Testing

Introduction

Last time we discussed the radial flow of water to a well, and introduced the Theis equation. We then showed how, given a transmissivity and storativity, the Theis equation can be used to calculate the amount of drawdown that would occur in the aquifer at some distance from a well (pumping at a constant rate) after some period of time. However, its greatest utility is in doing the opposite – measuring drawdowns in the aquifer, and using the data to estimate transmissivity and storativity of the formation. This chapter will focus on aquifer tests – tests that involve pumping wells and drawdown measurements and are used to estimate aquifer parameters.

Well Hydraulics – a Quick Review

Flow to a well is a special case because flow is radial – converging on a specific point in the aquifer (the well). To handle this mathematically, C.V. Theis stole an analytical solution to the equation that governs 2-D radial flow to a well…

\[
\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} + \frac{w}{T} \frac{\partial h}{\partial t} = \frac{S}{T} \frac{\partial h}{\partial t}
\]

(Equation 8-1)

where \( w \) = rate of vertical leakage or recharge to the aquifer [L·t\(^{-1}\)]

… and developed his eponymous equation, which is:

\[
s = \frac{Q}{4\pi T} W(u)
\]

(Equation 8-2)

where

\[
u = \frac{r^2 S}{4T t}
\]

(Equation 8-3)

and \( W(u) \) is the well function, which is:

\[
W(u) = -0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \frac{u^5}{5 \cdot 5!} - ...
\]

(Equation 8-4)

and the variables are defined as follows:

- \( s \) = drawdown [L], defined as the initial head minus the head at time \( t \)
- \( Q \) = pumping rate from the well [L\(^3\)/t]; constant throughout the period
- \( T \) = aquifer transmissivity (where \( T = K_b \)) [L\(^2\)/t]
- \( r \) = radius (distance from pumping well) [L]
- \( S \) = storativity (where \( S = S_b \)) [-]
- \( t \) = time after the start of pumping [t]

The Theis equation calculates drawdown \( s \) at some distance \( r \) from the pumping well at some time \( t \) after pumping began for a constant pumping rate \( Q \). This equation basically describes the cone of depression that forms around a pumping well. If we know \( T \) and \( S \), we can use this equation to directly calculate the drawdown at any distance from the well at any time after the start of pumping.

But the problem is how do we measure \( T \) and \( S \) in the first place? It turns out that the best way we have to ‘measure’ \( T \) and \( S \) in our aquifer is to pump water from a well at a constant rate, measure drawdown in the aquifer at various times after pumping starts, and then use these data and the Theis equation to calculate \( T \) and \( S \). Doing this is called an aquifer test.
Aquifer Tests and the Theis Curve-matching Method

The Basic Multi-well Aquifer Test

To collect the data needed to perform this sort of analysis, you need to do a multi-well aquifer test. To do this, you need:

- A pumping well with a pump that allows you to control the pump rate
- One or more observation wells close enough to “see” the influence of the pumping well
- Some means of measuring water levels in the observation wells at specific times throughout the course of the test; could be either automatic (pressure transducers & computer loggers) or manual (a couple of students with e-lines)
- At least 12 hours (preferably more)

The following is the general procedure for running a pump test:

- Measure the static water level in all the wells – the initial water level before pumping begins.
- Start the pump, and immediately begin taking water level measurements in the wells at various intervals (starting with something like every 30 seconds).
- As time progresses, the rate of drawdown will decrease, and measurements can be spaced farther apart.
- Continue pumping for some pre-designated period of time (generally something like 12, 18, or 24 hours) taking measurements at more and more widely spaced intervals.
- Shut off the pump and measure the recovery of water in the pumping well (measure water levels at increments of time until the water level approaches the original static water level).

At the end of the test, you will have the following data:

- A pump rate
- A distance to each observation well
- A table of time versus drawdown measurements for each observation well

The time vs. drawdown data, when plotted on a scatter plot and fit with a line, will result in a specific shape, and this shape will be different for different types of aquifers. These curves are used to get our estimates of T and S. But before we get into that, there are some general conceptual things that we need to cover concerning the drawdown response of different aquifers to pumping wells

Drawdown curves

When we talked about storage parameters, we discussed the concept of elastic storage. This is the response of a confined aquifer to pumping; pressure in the aquifer is released, the aquifer materials and the water expand, and water is released from storage into the well. In a perfectly confined aquifer, all the water pumped out through the well is released from elastic storage, with no interformational leakage, recharge from above, or changes in saturated thickness. This will result in a very specific curve on a plot of time vs. drawdown that looks like figure 8-1.

![Aquifer Tests and the Theis Curve-matching Method](image-url)
In figure 8-2, we see the same plot for a confined aquifer with a leaky confining layer. In this case, the water that is leaking through the confining layer is adding water to the aquifer, so the drawdown is not as much as it is in the fully confined case. The initial part of the curve is like a fully confined curve, but when the leakage becomes significant, the curve is affected.

A similar response would show up in an aquifer that was in connection with a constant head boundary, like a river or lake.

The other type of boundary that we talked about was the no-flow boundary. In an aquifer with a lateral no-flow boundary, there would be less water moving towards the well, and later-stage drawdowns would be higher (Figure 8-3).

Similar to the previous scenario, the early stage of drawdown matches the fully confined curve, and then deviates from it when the influence of the boundary becomes significant.

The unconfined aquifer is a more complex system, in that the drawdown response is affected by several processes (Figure 8-4).

1. The first stage represents release from elastic storage.
2. In the second stage, the depression of the water table creates vertical gradients in the groundwater; this is equivalent to water leaking in from above.
3. In the later stage, depression of the water table significantly reduces the saturated thickness, and therefore, the transmissivity.
Theis Method – Non-leaky (or Fully) Confined

The Theis method is outlined in detail in chapter 6 of Domenico and Schwartz (1998). The basic idea is to plot the time vs. drawdown data on log-log paper, then match the data to a graph of the type curve, which is a plot of 1/u vs. W(u). Once you match your data to the type curve, pick a match point on the two curves. You can pick any point, but it’s usually a good idea to pick a simple one like 1/u, W(u) = (1,1). Once you have a match point, take the value of t, s, u, and W(u) at that point, plug them into the Theis equation, and solve for S and T. It’s just that simple!

Leaky Confined Methods

As we previously saw, the drawdown curve for a leaky confined aquifer will not follow the same shape as a non-leaky confined aquifer; therefore, we cannot use it to match with our non-leaky type curve. Hantush and Jacob developed a curve-matching method that is similar to the Theis method but includes additional type curves and an additional term in the well function that accounts for the contribution of interformational leakage. Their equation is:

\[ s = \frac{Q}{4\pi T} W\left(\frac{u}{r/B}\right) \]  

(Equation 8-5)

The procedure is the same as the Theis curve matching, except that they have a different table of well function values.

Unconfined Aquifers

In an unconfined aquifer, the drawdown response has three separate parts to it:

- the early response is the release of elastic storage
- the middle response is dominated by dewatering of pores and gravity drainage
- the later response is also dominated by gravity drainage, but the diminished saturated thickness is increasing drawdown response.

Newman developed an even more complicated curve-matching method in which the well function is composed of three separate arguments, one for each of the drawdown stages. A description of the equations and the type curves is given in the text.

In terms of the exercises in the associated lab packet, we will not have to deal with the leaky or the unconfined situations. However, you need to be aware of the conceptual difference between the different situations.

Modifications to the Theis Method

The well function in the Theis equation is:

\[ W(u) = -0.5772 - \ln(u) + u - \frac{u^2}{2 \times 2!} + \frac{u^3}{3 \times 3!} - \frac{u^4}{4 \times 4!} + \frac{u^5}{5 \times 5!} - \ldots \]  

(Equation 8-6)

where

\[ u = \frac{r^2 S}{4 T t} \]  

(Equation 8-7)

Cooper and Jacob noted that, when u becomes small, everything to the right of the term ‘\ln(u)’ becomes negligible and drops out of the function. Dropping out this term reduced the well function to a simple term and reduced the Theis equation to:

\[ s = \frac{2.3Q}{4\pi T} \log \left(\frac{2.25Tt}{r^2 S}\right) \]  

(Equation 8-8)
This is called the **Cooper-Jacob simplification** of the Theis non-equilibrium equation. This means that, in certain situations, we can use this simplification rather than the more complex Theis equation.

So what would make \( u \) get small in the equation?

- increase \( t \), or
- decrease \( r \)

This means that the equation is valid for situations where \( t \) is very large (late in the drawdown period) or where \( r \) is very small. There are a couple of different ways that this can be used.

### Time-drawdown Method

If we have time-drawdown data in a single observation well over time, we can use the C-J simplification to estimate \( T \) and \( S \). The method is as follows:

- Plot time vs. drawdown on semi-log paper (with time on the log scale)
- Draw a straight line fit to the data, focusing more on the later data; extend the line through the drawdown = zero axis
- Choose two points on the line that are one log-cycle apart in time (e.g. 10 and 100, 1 and 10, 100 and 1000)
- Determine the change in drawdown over that interval (\( \Delta s \))
- Plug that into the following equation and solve for \( T \)

\[
T = \frac{2.3Q}{4\pi\Delta s}
\]  
(Equation 8-9)

- Determine the value of time where the line crosses the axis (\( t_0 \))
- Plug into the following equation and solve for \( S \)

\[
S = \frac{2.25Tt_0}{r^2}
\]  
(Equation 8-10)

**Voila!** Instant aquifer parameters!

### Distance Drawdown Method

This method uses two or more wells with drawdown measurements from one point in time to estimate \( T \) and \( S \). The method is as follows:

- Plot distance of each of your wells vs. drawdown on semi-log paper (with distance on the log scale)
- Draw a straight line fit to the data and extend the line through the drawdown = zero axis
- Choose two points on the line that are one log-cycle apart in distance (e.g. 10 and 100, 1 and 10, 100 and 1000)
- Determine the change in drawdown over that interval (\( \Delta s \))
- Plug that into the following equation and solve for \( T \)

\[
T = \frac{2.3Q}{4\pi\Delta s}
\]  
(Equation 8-11)

- Determine the value of distance where the line crosses the axis (\( r_0 \))
- Plug into the following equation and solve for \( S \)

\[
S = \frac{2.25Tt}{r_0^2}
\]  
(Equation 8-12)

This method is also described in more detail in Domenico and Schwartz (1998).
Single Well Drawdown Estimates

You can use the Cooper-Jacob simplification to directly estimate the drawdown in the pumping well using the radius of the well as \( r \). This will give you a basic estimate, but is not too reliable, since it assumes that the well is 100% efficient. Since well efficiency is never 100%, an estimate of drawdown in a well using the Cooper-Jacob simplification will underestimate the amount of drawdown in the well.

Well Efficiency

Well efficiency refers to the difference between the expected performance of a well and the actual performance. What it basically means is that the water pumped into a well will lose some amount of energy from turbulence in the well screen or filter pack, and this loss of energy will result in lower heads in the well.

Drawdown in a well is the result of energy loss; as energy is removed from the system, the heads (i.e., water levels) get lower. Head loss in a well is the result of two processes:

- Head loss in the aquifer (called formation loss \( s \)) due to the laminar flow of water to the well
- Head loss in the well itself due to turbulent flow through the well screen (called well loss \( s_w \))

Total drawdown in a well \( s_t \) is the sum of these two drawdowns, or

\[
S_t = S + s_w
\]  
(Equation 8-13)

and the well efficiency \( E \) is defined as

\[
E = \frac{S_{\text{calculated}}}{S_{\text{measured}}}
\]  
(Equation 8-14)

Since we are dealing with turbulent flow, the relationship between well loss and discharge is not nice and linear; therefore, a well does not have a single efficiency. Instead, the efficiency will change dependent upon the discharge (higher discharge will result in lower efficiency). Every well does, however, have a value called the well loss constant \( C \), which is a coefficient that relates well loss to discharge. Well loss is actually proportional to some power of the discharge that approaches 2; we can approximate it with:

\[
s_w = CQ^2
\]  
(Equation 8-15)

We can calculate the well loss constant for a well by comparing the expected drawdown in a well to the actual measured drawdown. If we have the well loss constant, then we can calculate well loss for any discharge rate from the aquifer.

The key thing to understand here is the concept of well loss from turbulent flow into the well screen. All wells have efficiencies that are less than 100; the lower the efficiency, the more energy it takes to get water out of the well.
Chapter 9 – More Aquifer Testing and Well Hydraulics

Introduction
(with a quick review of the last chapter)

The last two chapters discussed well hydraulics and aquifer testing. Multi-well aquifer tests, where we pump water from a well and observe the head changes in one or more other wells, are used to estimate transmissivity and storativity of an aquifer based on the Theis equation. Drawdown vs. time in the observation wells in a fully confined aquifer that satisfies all the assumptions of the Theis equation follows a specific curve on a graph that we called the type curve. In aquifers that are not fully confined, or are unconfined, or are influenced by a boundary, the drawdown vs. time curve will deviate from the type curve in ways that are specific to the type of aquifer. We also saw how Cooper and Jacob derived a simplification of the Theis equation that could be applied to certain situations.

The last thing we talked about was well efficiency, which basically refers to the difference between the expected drawdown in a pumping well and the actual measured drawdown in a well (which will be greater due to additional head loss in the well screen). In chapter, we will re-visit well efficiency, then discuss specific capacity and well yields, hydraulic boundaries and image wells, and then finish with some aquifer tests that can be performed in a single well.

Well Efficiency

Well efficiency refers to the difference between the expected performance of a well and the actual performance. What it basically means is that the water pumped into a well will lose some amount of energy from turbulence in the well screen or filter pack, and this loss of energy will result in lower heads in the well.

Formation Loss vs. Well Loss

Drawdown in a well is the result of energy loss; as energy is removed from the system, the heads (i.e., water levels) get lower. Head loss in a well is the result of two processes:

- Head loss in the aquifer (called formation loss ($s$)) due to the laminar flow of water to the well
- Head loss in the well itself due to turbulent flow through the well screen (called well loss ($s_w$))

Total drawdown in a well ($s_t$) is the sum of these two drawdowns, or

$$s_t = s + s_w$$  \hspace{1cm} (Equation 9-1)

and the well efficiency ($E$) is defined as

$$E = \frac{s_{\text{calculated}}}{s_{\text{measured}}}$$  \hspace{1cm} (Equation 9-2)

Since we are dealing with turbulent flow, the relationship between well loss and discharge is not nice and linear; therefore, a well does not have a single efficiency. Instead, the efficiency will change dependent upon the discharge (higher discharge will result in lower efficiency). Every well does, however, have a value called the well loss constant ($C$), which is a coefficient that relates well loss to discharge. Well loss is actually proportional to some power of the discharge that approaches 2; we can approximate it with:

$$s_w = CQ^2$$  \hspace{1cm} (Equation 9-3)

We can calculate the well loss constant for a well by comparing the expected drawdown in a well to the actual measured drawdown. If we have the well loss constant, then we can calculate well loss for any discharge rate from the aquifer.

The key thing to understand here is the concept of well loss from turbulent flow into the well screen. All wells have efficiencies that are less than 100; the lower the efficiency, the more energy it takes to get water out of the well.
Specific Capacity

When someone drills and constructs a well, they are interested in evaluating its performance in some way that it can be compared to other wells. In other words, it is desirable to have a way of quantitatively expressing the well yield, or how much water you can pump out of the well. This is generally expressed as the specific capacity, which is defined as the pumping rate per unit drawdown, and is calculated by dividing the pumping rate by the drawdown.

Since specific capacity is a function of the drawdown in a pumping well, and drawdown is the sum of the formation loss and the well loss, specific capacity is directly affected by the well efficiency. If we know $T$ and $S$, we can calculate a theoretical specific capacity using the Cooper-Jacob simplification of the Theis equation and the radius of the well for $r$, like so:

$$Q/s = \frac{4\pi T}{2.3\log\left(\frac{2.25Tt}{r^2S}\right)}$$

(Equation 9-4)

However, in any well there is a certain amount of well loss, so the measured drawdown will be greater than the calculated, and the measured specific capacity will be less than the calculated.

Thought question: If we use a single pumping well to calculate a transmissivity for our aquifer, and we assume that there is some well loss, the transmissivity that we measure will be different than the actual transmissivity in the aquifer. Will the transmissivity that we determine be higher than the real aquifer transmissivity, or will it be lower?

Boundaries and Image Wells

Superposition

Consider this now-familiar scenario: if we pump from a well at a constant rate for a certain amount of time, an observation well at some distance away (but not too far away) will experience a certain amount of drawdown. What happens to the observation well if we have more than one pumping well in the general vicinity (Figure 9-1)?

Each pumping well influences the observation well, and the resulting drawdown in the observation well will be greater. The effect of pumping from one well on the cone of depression developed by a second well is called well interference. Fortunately for us, all the equations that we are dealing with are basically linear, so it turns out that the total drawdown at the observation well is simply the sum of the drawdowns induced by each pumping well. This is the principal of superposition, that the effects of the two wells sum together to get a total drawdown.

Because of this, it is relatively easy to calculate the drawdown in an observation well influenced by multiple pumping wells, provided that you know $T$ and $S$ and are dealing with a specific point in time. All you do is calculate the drawdown from pumping well #1 at distance $r_1$, then calculate the drawdown from pumping well #2 at $r_2$, then sum the two drawdowns to get the total drawdown.
Use of Image Wells

This concept of superposition has a direct application in calculating the drawdown from multiple wells. It also has a second application in dealing with hydraulic boundaries in the aquifer. If you recall, we have two basic hydraulic boundaries – a no-flow boundary and a constant head boundary – and each of these has a specific influence on the drawdown response of the observation well (see Figures 8-2 and 8-3). The effect on drawdown from a boundary in an aquifer is basically the same as the effect of a second pumping well in an unbounded aquifer. Therefore, we can use something called image wells to calculate the effect of a boundary on drawdowns from a pumping well.

The basic idea behind image wells is that, if we know the location of the boundary and we know the transmissivity and storativity of our aquifer, we can mathematically place an imaginary pumping well in our aquifer an equal distance from the boundary, but on the opposite side of the boundary, and use those two wells and the principle of superposition to calculate the drawdowns in the aquifer (the same way we calculate drawdown from two wells pumping from the same aquifer). An example of a pumping well near a no-flow boundary is shown in figure 9-2.

The pumping well is located some distance $r_{\text{boundary}}$ away from the edge of the alluvial aquifer. The boundary created by the uplifted block of granite reduces the amount of water that is available to flow to the pumping well in exactly the same way as a second pumping well in an unbounded aquifer an equal distance from the boundary pumping at the same rate (Figure 9-3). So, if we want to calculate the drawdown at the observation well near the boundary, we would calculate the drawdown from the real well (“Pumping well”) at distance $r_{\text{real}}$, then calculate the drawdown from the imaginary well (“Image well”) at distance $r_{\text{image}}$, and add the two drawdowns together.

![Figure 9-2. Map view of pumping well at no-flow boundary showing image well (© Uliana, 2012).](image1)

![Figure 9-3. Cross-section of a system similar to that in figure 9-2 (© Uliana, 2012).](image2)
We can do the same thing with a constant head boundary, except that a constant head boundary is supplying additional water to the system and resulting in less drawdown. In that case, the image well is actually an injection well, and the additional drawdown from the image well is subtracted from the pumping well drawdown. (see Figures 9-4 and 9-5).

More information on image well theory can be found in Chapter 6 of Domenico and Schwartz (1998).

**Single Well and Piezometer Tests**

There are ways of using a single well to estimate hydraulic parameters for your aquifer.

**Pumped Well Recovery**

After a pump test is performed for the specified time period, the well is shut off and recover data (heads in the observation well at times after the pump is shut off) are collected. These data and the Cooper-Jacob simplification of the Theis equation can be used to calculate a transmissivity (but not a storativity).

**“Slug” Tests**

Multiple well pump tests are not always feasible – there may not be any observation wells, and it may cost too much to put new wells in. Or, it could be that you are dealing with a contaminated system, and everything you pump out of the well has to be hauled away and treated (more $$$). There are alternative methods that involve piezometers and a general set of tests called “slug” tests. These tests involve introducing or removing a known quantity of water (a “slug” of water) from a piezometer and measuring the time it takes to recover to the initial static water level.

In these tests, you generally measure the static water level, then remove or introduce your slug of water, take a measurement at the start of recovery (h₀), then measure the heads (h) at various times during the recovery period. The quantity h/h₀, sometimes called the residual drawdown, is then plotted against time and analyzed using one of various methods.

**Cooper-Bredehoeft-Papadopulous Method**

This method uses a set of type curves like the Theis method for multi-well tests.
- Plot \( h/h_0 \) and time on semi-log paper (\( h/h_0 \) on vertical arithmetic axis; time on horizontal log axis)
- Connect points with a best-fit curving line
- Match the data curve to the closest curve on the type curve graph and determine the value of \( \alpha \) for that curve
- Keeping the curves matched, determine the time value (\( t \)) on the field data curve that coincides with \( W = 1 \) (on the type curve graph).
- Solve for \( T \) using the following equation

\[
T = \frac{r_c^2}{t}
\]  

(Equation 9-5)

where

- \( r_c \) = radius of the well casing [L]
- \( t \) = time value that corresponds to \( W = 1 \) [t]

- Solve for \( S \) using the following equation

\[
S = \frac{r_c^2 \alpha}{r_s^2}
\]  

(Equation 9-6)

where

- \( r_c \) = radius of the well casing [L]
- \( r_s \) = radius of the well screen [L]
- \( \alpha \) = value associated with matched curve from the graph

This method will give you a transmissivity and a storativity.

**Hvorslev Method**

- Plot \( h/h_0 \) and time on semi-log paper (\( h/h_0 \) on vertical log axis; time on horizontal arithmetic axis)
- Connect points with a best-fit straight line
- Determine the value of \( T_o \) from the graph (defined as the time where \( h/h_0 = 0.37 \))
- Plug into the following equation:

\[
K = \frac{r^2 \ln \left( \frac{L}{r} \right)}{2LT_o^2}
\]  

(Equation 9-7)

where

- \( r \) = radius of the borehole
- \( L \) = length of the screened interval

Note that this method gives you the hydraulic conductivity, not the transmissivity.

**Potential Problems with Slug Tests**

When using a single well to evaluate the hydraulic parameters in an aquifer, the well loss and the influence of the sand pack and screened interval become significant. When performing a multi-well aquifer test with a pumping well and a single observation well, the drawdown response in the observation well reflects the transmissivity and storativity of the part of the aquifer between the two wells. In effect, we are measuring the properties of the aquifer in between the two wells. However, when using a piezometer to run a slug test, there is a possibility that the conductivity of the sand pack around the well screen may be so much different in conductivity than the formation that we are actually measuring the conductivity of the filter pack rather than the formation itself. Multi-well tests are always a more accurate representation of the aquifer; however, single well tests are sometimes our only choice.
Chapter 10 – Flow Systems and Regional Groundwater Flow

Introduction and Review

Let’s give a general review of the things that we’ve discussed to date. We started by considering the properties of water itself and presenting the hydrologic equation, which is also known as the conservation of mass or the continuity equation. Then we discussed the concept of fluid energy, how energy is distributed throughout a groundwater system, and how the distribution of fluid energy creates a hydraulic gradient that drives the flow of fluid through a porous medium. This is a fundamental aspect of hydrogeology – that groundwater discharge is driven by a hydraulic gradient, and that groundwater flows in the direction of decreasing fluid energy (i.e., decreasing hydraulic head). We then looked at the relationship between the discharge and the gradient, and we saw that the two are linearly proportional to each other, and that there is a constant of proportionality called the hydraulic conductivity (Darcy’s law). The hydraulic conductivity is a type of permeability, where permeability refers to a mathematical description of the ease with which a fluid flows through a porous media.

Next, we looked at the properties of the porous media itself. We discussed porosity, density, and moisture content, and the relationships between these properties. We then discussed permeability in more detail, and showed how permeability refers to the relationship between the discharge and the gradient, just like in many other physical processes (like heat flux, electrical current, and molecular diffusion). We looked in more detail at Darcy’s law and used it to calculate discharge through an aquifer as a volumetric discharge (Q), as a 1-dimensional discharge (q), and as an actual average linear velocity (v)

Next, we looked at the fundamental equations that govern the flow of groundwater in 3-dimensions. We presented the equation and showed how the equation is basically a more complex version of the continuity equation expressed as a partial differential equation. The left side of the equation is the divergence of flux and describes the I – O part of the continuity equation, resolved in three dimensions. The right side of the equation describes the ∆S/∆t part of the equation, in terms of the storage parameter and the changes in head with time. We discussed the storage parameters in detail, looking at the difference in the behavior of confined versus unconfined aquifers and the appropriate storage parameters for each. In confined aquifers, addition or removal of water is controlled by the elastic response of the aquifer, and is governed by the specific storage. In unconfined aquifers, addition or removal of water comes from dewatering the pore spaces, and is governed by the specific yield. We then looked at the time differential in the right side of the equation, and discussed the difference between steady state and transient flow. We also discussed heterogeneity and anisotropy, and looked at how those factors affect the relationship between flow lines and equipotentials on a flow net. And we also introduced the concept of boundary conditions in aquifers.

After we discussed the basic physics and mathematics behind groundwater flow, we considered a specific situation – radial flow to a pumping well. We discussed the concept of radial flow, then looked at the basic equation we use to describe the response of an aquifer to pumping, or the Theis equation. First, we used the Theis equation to calculate heads at some point in the aquifer in response to a well pumping at a constant rate. We developed a conceptual understanding of aquifer response to a pumping well, and looked at the difference in that response with different aquifers (confined, leaky-confined, and unconfined). Then, we talked about multi-well aquifer tests, and we used the Theis equation and the aquifer test data to estimate transmissivity and storativity in a aquifer. We also looked at a modification of the Theis equation, and then discussed well efficiency, specific capacity, and well yields. We then revisited the concept of boundary conditions and looked at the influence of boundary conditions on aquifer drawdowns. Finally, we discussed piezometer tests and single well aquifer tests, and talked about the advantages and limitations of those tests.

The next step is to expand our study and consideration up to the scale of whole aquifers, or flow systems. In this chapter, we will define flow systems, differentiate between local and regional flow systems, and discuss the ways that we actually study these systems.
Characteristics of Flow Systems

A groundwater flow system is a three-dimensional entity that has the following components:

- Boundary Conditions – has some physical dimensions and some real boundaries
- Recharge – some area where water is getting into the flow system
- Discharge – some area where water is exiting the system.

Let’s consider each of the three components.

Boundary conditions

As we said, flow systems are three-dimensional bodies and have boundaries. There are two basic boundaries, or boundary conditions, that we use to characterize the limits of our flow systems.

- **No Flow boundary**: There is no-flow across the boundary (h/x = 0). Examples:

  Permeable aquifer units could be in contact with low K crystalline rocks (Figure 10-1).

  ![Figure 10-1. No-flow boundary (© Uliana, 2001, 2012).](image)

Flow does not cross flowlines; wherever there are parallel flow lines, we have a **symmetry boundary** (Figure 10-2).

![Figure 10-2. No-flow symmetry boundary (© Uliana, 2001, 2012).](image)
Locally high groundwater levels can produce a groundwater divide; this is also a no-flow boundary (Figure 10-3).

**Constant Head**: the head does not change (e.g., river, lake). Examples:

When a non-flowing body of water (lake, pond) creates a constant head boundary, the shore of the lake represents a single equipotential line. Water is therefore flowing into the lake perpendicular to the shoreline (Figure 10-4).

With a river boundary, the heads vary along the boundary, but are still considered constant. Each point along the river represents a point on an equipotential (Figure 10-5).
Recharge

Recharge refers to the water that is entering a groundwater system. Areas where recharge is occurring are called recharge areas or recharge zones. There are several different ways that recharge gets into a flow system.

- Direct infiltration on the outcrop – some percentage of rainwater seeps into the ground and makes its way to the water table.
- Infiltration through the beds of losing streams or reservoirs – in some parts of the world (especially arid regions), this is the dominant form of recharge to any system.
- Interformational flow – usually flow through leaky confining layers; either where water is drawn in by drawdowns at wells or where underlying aquifers have significant overpressuring and are forcing water up.

Recharge is generally thought of as some percentage of precipitation. When it rains, some percentage of the water runs off on the surface, some evaporates back into the atmosphere, some is taken up by plants and animals (transpiration), and the remainder gets to the water table and recharges the aquifer. We can measure precipitation pretty well. We can also measure stream discharge and runoff, and get a good estimate of how much of a rainstorm ends up as runoff. We can also measure potential evaporation by placing a pan of water outside and seeing how long it takes to evaporate. However, it is more difficult to estimate transpiration and actual evaporation. Transpiration is difficult to quantify because any given area has a wide range of plants, and the overall transpiration is a function of the various percentages of each type of plant present in the area. Actual evaporation is hard to quantify because it depends upon how much water is actually sitting on the surface at any given time; plus the fact that evaporation rates during and after a rainstorm will not stay constant. Because of this, it is always difficult to put a number on recharge to any aquifer. Generally, it is accepted that annual recharge rates in most aquifers are somewhere between 1-10% of annual precipitation.

Discharge

There are several ways that water discharges from a flow system. They include:

- Discreet discharge to a spring or seep
- Discharge into a gaining stream or lake
- Pumping from a well
- Interformational flow (leakage through a confining layer)
- Evaporation – in some arid areas, this is the primary mechanism for discharge from the flow systems.

Discharge can also be hard to quantify, especially in areas dominated by well pumping or evaporation. Interformational flow is generally small compared to the other mechanisms. Springs and gaining streams can be gaged, and changes in flow across a certain area can be attributed to either recharge or discharge.

Local versus regional

When dealing with flow systems, we often differentiate between local and regional flow systems. For the most part, it is a matter of scale; local flow systems cover a much smaller area than regional systems. But we can establish some criteria to differentiate between the two.

Regional groundwater flow systems are defined by the following:

- Over a large area ($10^4 - 10^7$ km)
- Covering whole counties or states
- Contain several local flow systems (i.e., recharge and discharge areas within the overall system), and therefore…
- May have areas with local recharge as well as springs or other discharge features within the larger system

Often we can identify several different aquifers that are all connected by a regional groundwater system.
Heterogeneity and anisotropy

Heterogeneity refers to variations with respect to location. A parameter like permeability will vary from place to place in a flow system, depending upon the rock types and variations in sediment characteristics (like grain size). A common example of heterogeneity in a flow system is interbedded sand and clay layers in a fluvial depositional system.

Heterogeneity affects flow paths in two ways. One, if the interface between the different zones is parallel to the flow directions, water will tend to move in the path of least resistance. This affects the actual flow paths that water takes in a system (Figure 10-6).

If flow paths are crossing the interface, the flow directions will be refracted by some angle that depends on the difference in permeability between the two zones. The result of this in real flow systems is that flow in the low-permeability units tends to be vertical, while flow in the aquifers tends to be horizontal (see the figures at the end of Chapter 6 – More on Flow Equations).

Anisotropy refers to variations with respect to direction. Layering is one form of anisotropy; in a layered system of interbedded sand and clay, the horizontal permeability will be much greater than the vertical permeability (see Figure 11-7). Anisotropy can also result from sedimentary structures formed during deposition, both on a large scale (like channel sands interspersed with overbank deposits in a fluvial system) and on a smaller scale (cross-beds, graded bedding). More significantly, geologic structures like fractures are frequently responsible for anisotropic aquifers.

As we saw in Chapter 6 – More on Flow Equations (Figure 6-3), if the direction of maximum permeability is not parallel to the gradient, the flow paths will be deflected and will not follow the gradient. In highly anisotropic aquifers, a groundwater map showing the equipotentials will not necessarily indicate the primary flow directions. For example, in the Edwards aquifer of central Texas the gradient points towards the Gulf...
Coast. However, extensive fracturing associated with the Balcones Fault Zone, and later karstification, has created an anisotropy that has deflected the actual flow paths so that they are almost parallel to the equipotentials. This is a concern when dealing with real aquifers that are extensively fractured.

**Relationship between topography and flow systems**

King (1899) and Hubbert (1940) noted the relationship between topography and groundwater flow patterns. Both noticed that the water table tends to be a subdued replica of the topography (Figure 10-8).

In 1963, a guy named Toth developed a computer model, using the same groundwater flow equations that we've been dealing with these past few weeks. This was a big deal because it was one of the first numerical computer groundwater models and it paved the way for a lot of later groundwater modeling techniques. His model looked at how flow systems are related to topography. He set up his model as a 2-dimensional cross section, and used some mathematical equations to simulate uneven topography on the surface. When he varied that topography, he noted that, in some places, smaller local flow systems would develop within the larger regional flow systems. This was the first mathematical 'evidence' that these types of systems exist, and it confirmed the finding of King and Hubbert.
Chapter 11 – Studying Flow Systems

Introduction

The last chapter discussed the characteristics of flow systems and discussed the concepts behind local and regional systems. Basically, a flow system has some boundaries that constrain the system, some source (or sources) of recharge to the system, and some mechanism (or mechanisms) for discharge from the system. The boundaries that constrain the system can be physical boundaries that actually exist (like a constant head boundary at a lake) or they could be boundaries that we impose upon the system so that we can study a certain part of the system (like a no-flow symmetry boundary). Recharge can be from a number of sources, but generally is expressed as some percentage of average precipitation (usually somewhere between 1 and 10%). Discharge includes natural forms (like springs, seepage into rivers, and evaporation) as well as groundwater withdrawal through wells. In many aquifers, wells are the primary source of groundwater withdrawal.

The last chapter also discussed the difference between local and regional flow systems, the relationship between topography and groundwater flow, and how Tóth (1963) developed one of the first computer groundwater models that showed how changing topography can create local flow systems within larger regional systems. We also revisited the concepts of heterogeneity and anisotropy, and talked about how they affect flow systems.

However, like anything else in geology, it is one thing to understand the theory and the basic concepts, but quite another to actually study flow systems in reality. There is so little that we can actually observe in groundwater systems (like in most geologic systems) that we have to collect various types of data that will allow us to make inferences about what is going on in the subsurface. In this chapter, we will discuss the methods that we use to study groundwater flow systems, talk about the types of data that we can collect, and discuss the ways that we interpret those data.

How Do We Study Regional Flow Systems?

The basic idea behind studying regional flow systems is to:

- Determine the boundary conditions and the geometric limits of the system
- Identify the mechanism(s) of recharge, the magnitude of recharge, and the areas where recharge is occurring
- Identify the mechanism(s) of discharge, the magnitude of discharge, and the areas where discharge is occurring
- Identify the parameters that characterize the system (T, S, etc.) and determine how they vary throughout the system

Simple, right? Well… not really. The first problem is that we are dealing with groundwater, and the vast majority of the system is underground. The actual volume of an aquifer that we can observe directly is an incredibly small percentage of the total volume of the aquifer. In addition, there is no single method for identifying boundary conditions, or determining recharge, or determining discharge.

In general, when studying aquifers and regional flow systems, we gather as much data as we can from as wide a variety of sources as possible, and try to identify some common direction in which all those data point. In addition to more traditional hydrogeologic methods, like aquifer tests and groundwater maps, we frequently use geochemical data, surface features and topographic information, and our basic understanding of geology to help us study flow systems.

The ultimate goal should be to develop a conceptual model of the flow system that we are studying: a model in our heads that describes the boundary conditions, the fluxes through the system, and the parameters that characterize the system. This conceptual model can then be tested by gathering more data, or by using simple mathematical calculations or more complex computer models.
What Data Can We Gather?

Surface Features
- Topography: can indicate recharge areas and give general indications of flow system boundaries (not applicable when dealing with confined aquifers)
- Surface water – lakes and rivers
  - discharge or recharge?
- Springs: location, discharge rates, chemistry

Hydraulic Heads
- Distribution of heads suggest flow paths and probable recharge/discharge areas

Water Balance
- Compare estimates of recharge…
  - Precipitation/evaporation data
  - Stream losses
  - to estimates of discharge (I – O)

Temperature Gradients
- Advection (i.e., flowing groundwater) can change temperature patterns in the subsurface
- Anomalous gradients can indicate vertical flow of groundwater

Groundwater Chemistry
- Not just contaminants; groundwater chemistry is controlled by interactions of water with rocks/soil and by mixing of waters
- We can use chemistry to infer groundwater flow paths
- Piper/stiff diagrams
- Geochemical models: simulate groundwater changes, then compare actual data

Isotopes in Groundwater
- Stable (hydrogen, oxygen); Radiogenic (carbon, strontium)
- Can be used to trace groundwater flow paths
- Also indicate recharge areas and conditions
- Dating of groundwater also possible

Geology
- We need an understanding of the stratigraphy and structure of an area to truly understand the hydrogeology
Chapter 12 – Flow Modeling

Introduction

The word ‘model’ has come up time and time again in this class, both as a noun (“… develop a conceptual model…”) and as a verb (“… use an equation to model a system…”). This word means a lot of different things to a lot of different people; to some, it means a computer program that simulates some natural process, to others it a scaled-down physical representation of a thing that exists in nature (like a model car or a model solar system used in a classroom), and to some it means a tall, attractive, exotic-looking people that display the latest clothing styles on runways in Paris. We won’t be dealing with the latter in this chapter. However, the concept of ‘models’ is something that has applications in all the sciences, and it is important that we define what it means in general and what it means to hydrogeologists in particular.

Models: A General Definition and Specific Details

There are a lot of different definitions of the word ‘model’; so many that I came up with my own. A model is a simplified representation of an object, structure or process that cannot be studied directly because it is too large, too small, or too complicated, or because direct study or observation is, for other reasons, not possible or feasible.

One of the keys to this definition is that we can represent a process as well as a structure or object. We traditionally think of models in the sense of things that represent the structure or appearance of something, like a plastic model airplane or a stick model of a benzene molecule. Model airplanes can be very sophisticated in their representation of the appearance and detail of a real airplane – everything is scaled down correctly, and the details can even include the rivets on the wing and the instrumentation in the cockpit (and try not to giggle when I say “cockpit”). However, the model just looks like a plane; it can’t actually act like a plane (i.e., it can’t fly). The model of an organic molecule is the same way – it may have the correct bond angles and the relative size of the atoms is correct, but it does not exhibit the same behavior as a real molecule – it just sits there. These types of models are called geometric models, because they depict the geometry (i.e., structure, appearance, distribution, etc.) of whatever it is they represent. Some examples of geometric models are:

- plastic toy figures of WWII fighter planes
- blueprints
- geologic maps and cross sections
- molecular symbols and molecular models (Figure 12-1)

It is of more importance to us as scientists and hydrogeologists to be able to develop process models, or models that simulate physical processes. These models have to contain some component of the geometry of the system, but they also have some component that describes how the system ‘acts’ (i.e., how it responds to changes in certain conditions, how it changes with time, etc.) This component can be an actual physical thing, or it could be a mathematical component (i.e., an equation or a set of equations).

Process models can simulate things like:

- Global climate, climate change, and climate variations
  - Simulate the response of global climate to changes in atmospheric CO₂
  - Simulate the El Nino Southern Oscillation
- Stress and strain
  - Simulate stresses in a bridge or building foundation
  - Simulate strain in deformed geologic units
- Watersheds
  - Simulate flooding in response to large storms
  - Calculate flow rates at various points in watershed
- Aquifers and groundwater flow systems
  - Simulate changes in head with response to pumping
  - Calculate well yields

Why do we use process models?

There are a variety of reasons for developing a process model of a system. The main reasons are outlined below.

**Make Predictions**

Probably the obvious reason is that models are used to make predictions, either about how a system will change in the future or how the system might respond to some change or some stress that we apply to it. This is the main objective of the associated laboratory semester project—to develop a model of a specific study area and use it to estimate how much water can be pumped from the ground and what the resulting change in head would be. In this case, the pumping is the stress that we apply to the model, and the head changes are the response of the model. It is probably a safe assumption that most models are developed for this purpose.

**Test Hypotheses and Investigate Processes**

In some cases, we may not be sure why a natural process behaves the way it does, and a model can help us constrain our hypotheses concerning that behavior. For example, we may be studying the structure of a set of folded and faulted rocks. We don’t know exactly what stresses caused the strain that we observe, so we make some hypotheses about compression and tension and the principle stress directions. We can then put together a model that simulates those stresses, and compare the model results to the actual structure of the rock. If the model results don’t match, then we need to refine or reformulate our hypotheses. If the model results do match, it supports our hypotheses (however, it doesn’t prove anything—we’ll discuss that in a minute).

**Model Sensitivity**

Another use of models is to help the investigator identify areas where more data is needed. Models can be used to test the sensitivity of the area to certain parameters, and a model may show that the variations of one parameter don’t make a big difference, but another one does. For example, we may be studying an aquifer and we may want to calculate how much drawdown we will see around a pumping well. If we have two sets of aquifer tests with two transmissivity values that are almost an order of magnitude apart, we may wonder if we need more transmissivity data to calculate drawdowns throughout the study area. A model may tell us that variations in transmissivity do not significantly affect the model results, so we can therefore go ahead and use the data that we have rather than go to the trouble and expense of collecting more data.

**Augment Lack of Data**

Another reason to develop a process model is to use the model to estimate parameters in areas where data is limited. For example, in the CenTex project (in the associated lab manual), we have two transmissivity values from two different pump tests, the two values are an order of magnitude apart, and we need to distribute those values throughout the study area if we are going to predict drawdowns throughout the study area. We hypothesize that transmissivity is directly correlated to percent of sand in the aquifer, and we may want to distribute transmissivity accordingly. We can put together a model that simulates that distribution and match it to the water levels that we have in our wells; if they match pretty well, then the model suggests that our hypothesis about the distribution is probably pretty good.
Estimate Parameters (Inverse Models)

If we know a lot about our system, but have uncertainties concerning a specific parameter, we can use a model to estimate that parameter by basically plugging in the information that we know and solving for what we don’t know. These are called inverse models, because the procedure is basically opposite the way things are normally done. Figures 12-2 and 12-3 are an example of a really simple inverse model, developed as part of the author’s dissertation, which estimates transmissivity in a regional flow system.

Doing the calculations with the system outlined in figure 12-3, and recharge values ranging from 0.5 to 5% of annual precipitation, results in transmissivity values ranging from 1.07 m²/s to 10.7 m²/s. These values are within the range of values estimated from pump tests and other methods.

Specific Types of Groundwater Models

We have defined the word model and have talked about the reasons we have for using models. Now, we will discuss in more detail the various types of models, concentrating on models used to simulate groundwater flow.
Qualitative Models

Simple groundwater contour maps can primarily be thought of as geometric models, but they also give us qualitative information about the processes. We can look at a groundwater contour map and infer the direction of groundwater flow, and as we saw in the flow net exercise on the midterm, we can use maps and flow nets to identify heterogeneities and anisotropies. The groundwater flow map for the CenTex project indicates both direction of flow and heterogeneities in the system.

Physical or Analog Models

People have actually developed physical models that represent fluid flow, setting up tanks full of sand or other sediments and moving water through the tanks to simulate groundwater flow. Physical models made out of layers of sand or clay have also been used extensively in structural geology problems. Another physical model simulating fluid flow used two glass plates sandwiched together with small channels etched on the surface of the plates.

Analog models use the similarity between Darcy’s law and other physical processes to model fluid flow. The most popular way of doing this is with electric models in which the flow of current represents the flow of groundwater, the changes in voltage represent the changes in head, and the resistance of the conductors represents the hydraulic conductivity.

\[ q = K \frac{dh}{dl} \]  
\[ I = \frac{1}{R} V \]

(Mathematical Models)

Any equation that represents some physical process is a model of that process. Darcy’s law is a model that represents the flow of groundwater in one dimension. Because it is a model, it tells us something about how the system behaves (flow is linearly proportional to the hydraulic gradient) and it can be used to make predictions about the system (calculating travel times and flow velocities).

We spent a good deal of this book talking about the basic equations that govern the flow of groundwater in a three dimensional, transient system. We can develop solutions to this equation and use them to develop models that simulate groundwater flow in flow systems and aquifers.

The basic groundwater flow equation is:

\[ \frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} \right) = S \frac{\partial h}{\partial t} \]  

(Equation 12-3)

This equation describes the flow of groundwater and change in storage with time at a single point in a flow system. The problem with using this equation is that we have an infinite number of points in any system (and therefore an infinite number of equations to solve), and the heads and fluxes at each point are dependent upon the heads and fluxes at all the surrounding points. We can’t simultaneously solve an infinite number of equations, so we have to simplify the system in some way and then use that simplification to solve the problem and model the system. There are a large number of methods for solving equations like these, but they are variations of two basic methods of solving partial differential equations.

Analytical Solutions

Sometimes we can take a specific scenario and develop some assumptions that simplify the system, and use those assumptions to develop an analytical solution to the equation. The Theis equation is an example of this – Theis considered a specific situation (radial flow to a well) with a number of simplifying assumptions.
(radial flow, constant thickness, infinite extent, etc.), and derived an equation that directly solves for the
drawdown in the aquifer at some distance from a pumping well and at some time after pumping started.

Analytical solutions are advantageous because they are easy to use (since they can be solved directly) and
give unambiguous results. However, they are limited because they require simplification of the systems,
and so they are not always applicable where we want to use them. Also, we may have a specific situation for
which we want to derive a solution, but the situation may be so complex that a direct solution isn’t possible.

**Numerical Solutions**

If an analytical solution is not possible, we may have to simplify the system in another way and develop an
approximation called a **numerical solution**. The basic idea behind a numerical solution is that you first
determine your boundary conditions, then reduce the infinite number of points (and corresponding equations)
in the system to a finite number of points, and then use an iterative method to solve the equations at those
points.

The process of reducing the system to a finite number of points is called **discretization**; i.e., making a
continuous function into a discrete function. This process is illustrated in figure 12-4.

![Figure 12-4. a) Alluvial aquifer, bounded by river and an impermeable boundary; b) discretization of the alluvial aquifer (© Uliana, 2001, 2012).](image)

In this example, we have taken an alluvial aquifer that contains an infinite number of points and discretized it
by superimposing a regular grid that consists of 48 points, or **nodes**. Each point contains a version of the
groundwater flow equation, solved for head, and each point is calculated based on the values of head at the
surrounding points. We have also assigned boundary conditions based on the physical boundaries of the
model (the river and the granite fault block) and based on the extent of our study (the symmetry boundaries).

![Figure 12-5. the value at any node is dependent upon the values at the surrounding nodes (© Uliana, 2001).](image)

Once we have this model put together, with the boundaries designated and the equations assigned to each node, we
assign an initial value of head to each node and we solve the equations. However, there is a little complication here;
the value of head at each node is dependent upon the value at the four surrounding nodes (Figure 12-5). So, if we
calculate the value at a single node based on the surrounding nodes, then go to the next node and calculate the
value, we have changed the second value and so have to re-calculate the first value. But by recalculating the first
value, we have to go back and recalculate the second value and so on and so on and so on…
This is where the numerical part of the method comes in. What we do is we start at one corner of the model, calculate a head at that node, then systematically move across the row and calculate the value at each node. Then move to the next row and do the same. When all of the values are calculated, you record the values at each node, then start over.

Once you’ve gone through the grid a second time, you compare the values from the second set of calculations to those in the first set of calculations and look at the difference between them. Each set of calculations is called an iteration, so you compare the results of the second iteration to the results of the first. Then do a third iteration, and compare those results to the second. Keep doing that, comparing the results of each iteration with the previous, and monitoring how the difference changes from iteration to iteration. What should happen is that the difference will get smaller and smaller with each iteration until it reaches some point where the changes are so small that they are negligible. At this point, we say that the model has converged, and we have an approximate solution to the model. The model can converge in as little as 2 or 3 iterations, or it could take hundreds or thousands of iterations. Sometimes, mathematical instability can occur and the model never converges.

This iterative process is where the importance of computers comes in. A person, using pen and paper and a calculator, could take years to calculate and solve a simple model with, say, 10 nodes. These models were not feasible until we developed computers that can do lots of calculations in incredibly short periods of time. Computing power is so great now that basic desktop computers can do very sophisticated models in short periods of time. However, the processing time can become significant if you put a lot of detail into the model.

When discretizing a system, you need to decide how much detail you want to put into the model, keeping in mind how much detail you expect to get out of the model. The more points you use, the more detailed the model will get; but the more computationally intensive the model will be. Also, more complexity increases the possibility of numerical instability and a model that won’t converge. In many cases, simpler models are better because there is less estimation and uncertainty, and frequently they tell us the information that we want.

The Method of Finite Differences

The equation that we put into each node is a finite version of the groundwater flow equation shown on page 4. In other words, the equation on page 4 represents the flow at an infinitesimal point, a point with no dimensions. When we put a finite number of nodes in our model, we have to transform the equation so that it takes into account the distance from one node to the next. There are a number of mathematical methods that we can use to do this; the one used in most groundwater models is the method of finite differences.

We won’t go into the derivation of the finite difference equations in too much detail here; but basically, the equation breaks down to the value of head at each node is the average of the heads at the surrounding nodes. If our system is heterogeneous, or isotropic, or if the spacing of our nodes is different in the x and y directions, it gets a little more complicated than that; for our purposes, we just need to know that the value at each node is dependent upon the four surrounding nodes.

A few things about finite difference models:

- Boundary conditions must be entered into the grid. The two main boundaries are no flow and constant head. For a constant head boundary, simply assign the head value to the node.

For a no-flow boundary, you treat it similar to an image well problem; you have an imaginary node equal to the opposite node (Figure 12-6)
- Each node requires a hydraulic conductivity and a storage parameter (specific yield if unconfined, specific storage if confined).
- Recharge can be added to each node; mathematically you simply add a volume of water to the node per time, and the storage parameter calculates how much head change results.
- Pumping wells can be put at any node; mathematically they act just like recharge, but in reverse.
- Must have a regular grid – column width and row height can vary, but individual cells in the grid cannot have variable spacing (Figure 12-7).

![Figure 12-7. The grid on the left is an appropriate finite difference grid; the one on the right is not (© Uliana, 2001).](image)

### Constructing Finite Difference Models: an Introduction to MODFLOW

A finite difference model is a set of computer code that does the calculations and records the output. A finite difference model code requires the following:

- Some means of inputting the grid nodes and the parameters (K, S_s) at each node
- An algorithm that
  - Calculates the head at each node and records that value
  - Runs through the whole grid for each iteration
  - Compares the results from one iteration to the next
  - Decides if the model has converged or if more iterations are necessary
- Some means of writing the final head values to a text file for output

Constructing simple finite difference models and writing a computer code to do the calculations is not too difficult. However, models can get very complex in a very short period of time, and the time spent writing the code can increase dramatically.

It used to be that, if you wanted to do a computer groundwater model, you had to build it from the ground up for your specific situation. In the late 70’s, people started writing model codes that were flexible and could be applied to a variety of situations. Then in 1984, two guys at the USGS named McDonald and Harbaugh developed a model code called MODFLOW which was the first published model code that could be widely used.

The key to their model was that, rather than set up a single computer program with all of the steps outlined above, they set up their software as a series of subroutines, or modules, and each module is associated with a specific part of a finite difference model. For example, they have a pumping well module, a recharge module, a river boundary module, etc. The user is still responsible for putting together the input files and setting up the grid, but then they can choose the modules that are appropriate for their model and construct it accordingly. This, along with the fact that their software was free, resulted in widespread distribution of the model and an almost universal acceptance of the program.
However, the program that they set up is still pretty unwieldy – it is extremely time consuming and difficult to put together the input files (these are huge text files that require specific line spacing, etc., and the potential for human error is great). Therefore, non-USGS affiliated companies started developing and selling **preprocessor** software that use GUI interfaces and special programs that aid in grid construction and parameter distribution. These are then linked directly to MODFLOW, so the model grid and parameters go right into the MODFLOW algorithm. They also developed **post-processor** programs that take the output and make nice fancy pictures with it. These programs are in widespread use today, and they give any individual with a standard PC computer the ability to generate and run fairly complex groundwater models.
Chapter 13 – Mass Transport

Introduction

There are two aspects of water chemistry that interest hydrogeologists. The first is the problem of groundwater contamination and the spread of contamination in the subsurface. These contaminants are generally organic compounds (chemical compounds that contain carbon) and frequently are derived from petroleum. This is a problem that we are all probably aware of to a certain extent, and a lot of research and work has been done trying to understand the processes and to characterize and clean up contaminated sites. It can be an extremely complicated problem, because there are a number of processes that are moving the contaminants and many possible reactions that create, change, and break down contaminants.

The second deals with the natural chemistry of groundwater systems and the interaction of groundwater with solid mineral phases and organic matter. Water, known as the ‘universal solvent’, reacts with and dissolves most anything it encounters. The rocks and sediments that make up an aquifer (primarily inorganic compounds) will impart a chemical signature to the water, in the form of a specific set of dissolved ions. This is useful to us because we can use the groundwater chemistry to infer where the water is coming from and to link the groundwater to specific geologic units.

We will start by discussing the processes that move particles (dissolved or suspended) through groundwater systems. Then we will discuss the reactions that affect the concentrations of various contaminants and naturally-occurring dissolved minerals.

Mass Transport in Groundwater

When we use the term mass, we mean anything in the water that isn’t water. Mass occurs in water as dissolved ions, molecules, colloidal particles, or other suspended particles. Mass transport refers to the movement of these particles in groundwater systems. There are two basic processes that move particles through groundwater systems. These are advection and dispersion.

Advection

As groundwater flows through a porous media, it will carry particles along with it. This process is called advection.

In figure 13-1A, we see purely advective mass transport from a continuous point source of contamination. The cross hatched area signifies the source of the contamination (e.g., a settling pond, a leaking underground storage tank), and the gray area represents the contamination in the groundwater at some time (t) after the contamination release began. The concentration of contamination is constant throughout the gray area.

In figure 13-1B, we see purely advective mass transport from a single volume of contamination released from a point source at a specific point in time. The cross hatched area represents the location of the contamination release (e.g., a spill from an oil tanker), and the gray area represents the contamination in the groundwater at some time (t) after the release of the contaminant. The concentration of the contaminant in the gray area is the same as it was at the initial release.
Advective transport is simply a function of the velocity of the groundwater. If we assume that advective transport is the only process moving the contaminant, and we know the hydraulic conductivity and effective porosity of our aquifer, we can calculate the travel time, or the time necessary for the contaminant to move a specific distance (L), using the following procedure:

- Calculate the average linear velocity using Darcy’s law and the effective porosity

\[
\bar{v} = \frac{K}{\phi_{\text{eff}}} \frac{\partial h}{\partial x}
\]

(Equation 13-1)

where

\(\bar{v}\) = average linear velocity [L/t]

K = hydraulic conductivity [L/t]

\(\phi_{\text{eff}}\) = effective porosity [-]

\(\partial h/\partial x\) = hydraulic gradient [-]

- Divide the distance (L) by the average linear velocity to get travel time

Thought question: This assumes that the mass itself does not affect or influence the pattern of flow. Is there a possibility that the contaminant may affect flow?

Breakthrough curves

A good way of visualizing and understanding mass transport processes is through the use of breakthrough curves. Breakthrough curves are basically graphs that show the relationship between concentration of the contaminant and either time after release or distance from the point of release. Breakthrough curves for the situations in figures 13-1A and 13-1B are given in figures 13-2 and 13-3, respectively.

Figure 13-2. Breakthrough curve for purely advective transport with a continuous source (see Figure 13-1A) (© Uliana, 2001, 2012).
Figure 13-3. Breakthrough curve for purely advective transport with a single volume source (see Figure 13-1B) © Uliana, 2001, 2012.

The vertical axis on the breakthrough curves represents the concentration of the contaminant \( C \) divided by the original concentration \( C_o \). This quantity represents the fraction of contamination in the water; where \( C = C_o \), \( C/C_o = 1 \) and the contamination is 100% of the original contamination.

Concentration of contamination is usually expressed as a mass per volume of water (e.g., mg/L) or as a dimensionless mass per mass (e.g., parts per million). Using \( C/C_o \) is a way of expressing the concentration as a fraction or percentage of the total, and we use it here to illustrate the basic concepts of contaminant transport.

**Dispersion**

In real groundwater systems, advection is usually responsible for moving the largest quantity of mass. However, as the contaminant moves through the subsurface, several processes act on it and cause the contamination to spread out, resulting in variations in the concentration throughout the contaminated area. This spreading of particles in the subsurface is called dispersion, or sometimes hydrodynamic dispersion. Dispersion parallel to the flow vector is called longitudinal dispersion, while dispersion perpendicular to the flow vector is called transverse dispersion. Longitudinal dispersion spreads out the contaminant in the direction of flow, while transverse dispersion spreads it out laterally (to the side) as the contaminant moves with flow.

Dispersion is caused by two main processes. The first is molecular diffusion, which is the movement of dissolved solutes under the influence of a concentration gradient (i.e., from areas of high concentration to areas of low concentration). This process is driven by the random motions of the dissolved ions and is a function of the temperature of the water (higher temperatures = more energetic and lively ions = more diffusion). We can measure a diffusion coefficient – a parameter that relates concentration to the rate of diffusion.

The second process involved in dispersion of contaminants in groundwater is called mechanical dispersion. The longitudinal component of this is related to the variations in water velocity from point to point in a groundwater system (remember when we discussed specific discharge vs. the average linear velocity vs. the true groundwater velocity at any given point?) The transverse component is due to the tortuous paths taken by groundwater as it moves around sand grains and through pore spaces. Figures 13-4A and 13-4B show the effects of dispersion on contaminant transport from a continuous source and from a single pulse of contamination.
These figures show the same situations as figure 13-1 with longitudinal and lateral dispersion. In figure 13-4A, we see advective mass transport with dispersion from a continuous point source of contamination. The cross hatched area signifies the source of the contamination (e.g., a settling pond, a leaking underground storage tank), and the lines of equal concentration represent the fraction of contamination remaining throughout the system at some time (t) after the release of contamination.

In figure 13-4B, we see advective mass transport with dispersion from a single volume of contamination released from a point source at a specific point in time. The cross hatched area represents the original location of the contamination release (e.g., a spill from an oil tanker), and the lines of equal concentration represent the fraction of contamination remaining throughout the system at time = t.

The breakthrough curves for figures 13-4A and 13-4B are shown in figures 13-5 and 13-6, respectively.
Thought question: how does the dispersivity affect the shape of the breakthrough curve (i.e., what would a curve with a large dispersivity look like versus one with a small dispersivity)?

Reactions and Retardation

Some solutes are stable and do not undergo reactions in the subsurface. These solutes are called conservative solutes. An example is the chloride ion – chloride doesn’t stick to anything, it doesn’t undergo biological reactions involving micro-organisms in the subsurface, and is usually one of the last ions to precipitation out of groundwater. Not all solutes and contaminants are like that; in fact, most solutes and contaminants are reactive in the subsurface. This complicates our understanding of mass transport because, in addition to dilution of the contaminant from dispersion, the overall concentration of the solute or contaminant in the system is changing with time.

Some of the reactions do not involve degradation of the contaminant; they simply slow down the overall movement of the contaminant through the groundwater. This process is called retardation. Most retardation reactions involve sorption, which is a general term that describes how one thing sticks to another.

- Absorption: a contaminant particle is absorbed into the volume of something else (see Figure 13-7). An example: most organic compounds have high solubility in other organic compounds but have low solubility in water; organic contaminants in groundwater can basically “dissolve” into organic matter in soil or subsurface sediments

- Adsorption: a contaminant particle sticks onto the surface of something else (see Figure 13-7). Basically the same as absorption, but it’s a surface phenomenon. This happens with ions and clays; clays have charges on their surfaces, and this electrostatic attraction can grab ions out of solution.
Clays have very large surface area per volume, and tend to have significant electrostatic charge on the surface. In clays, the negatively-charged sites are more abundant than positive (positives tend to occur on the edges of the minerals). Divalent cations (cations with a +2 charge, like calcium and magnesium) are more strongly attracted to the clays than monovalent cations (cations with a +1 charge, like sodium). In some groundwater systems, divalent cations in solution will be pulled out by the clays and replaced with sodium ions. This process is called **cation exchange**.

**Capture Zone Analysis**

One way of dealing with contamination in the subsurface is through **pump-and-treat** remediation systems. These involve pumping water out through a well, treating it, and discharging the treated water back into the subsurface. When a well is pumped, there is a zone around the pumping well called a **zone of capture**. This zone basically corresponds to the cone of depression, and all the water within the zone of capture flows towards the well. In an aquifer with a perfectly horizontal potentiometric surface, the zone of capture is a circle (or ellipse, if the aquifer is anisotropic). Real potentiometric surfaces are generally sloping, so the capture zone will be an elongated area that extends slightly down-gradient of the well as well as in an up-gradient direction. Figure 13-8 shows a cross-sectional and a map view of a capture zone in a homogeneous, isotropic aquifer.

![Figure 13-8. A) cross-sectional view of a capture zone, showing the original potentiometric surface and the stagnation point; B) map view of capture zone showing the divide that delineates the zone (© Uliana, 2002, 2012).](image)

The pumping well will reverse the gradient for a short distance down gradient of the well. Downgradient of the well is a **stagnation point**, which marks the point where the natural gradient is affected by the pumping well. Supposedly this marks a point in the aquifer where water is not flowing (hence the name).
Chapter 14 – Aqueous Geochemistry

Introduction

Mass can exist in the subsurface in a number of different phases. Mass exists in a solid phase, made up of either the minerals that make up the aquifer or suspended particles moving through the aquifer. Mass also can exist as a separate liquid phase; for example, many organic contaminants do not mix with water very well and stay in a liquid state in the subsurface. Mass can occur as a gas phase; most soils contain CO$_2$ in their pores, and bubbles of gas can occur in the saturated zone of aquifers. Finally, mass can exist as a dissolved phase. All of these phases are significant parts of aqueous geochemical systems, however, hydrogeologists are mostly concerned with the dissolved phase, and lump the other three phases together as sources for dissolved species in the water.

In this chapter, we will focus on mass as dissolved phases in groundwater, and consider any solid, liquid, or gas phases together as a separate phase that reacts with the groundwater. Groundwater is constantly reacting with the other phases in the subsurface. These reactions are responsible for the redistribution of mass in the subsurface and are what control the chemistry of the groundwater.

First, A Little Background Chemistry

This will be review for most people, but it is generally a good idea to make sure everyone understands the terminology and is on the same page with the concepts.

Basic Chemistry Concepts and Definitions

Atoms and Elements are the basic building blocks of nature; these are the things described the periodic table (attached). Each element has a symbol associated with it (e.g., carbon is C, potassium is K, neodymium is Nd). Atoms are made of the following smaller particles (Figure 14-1):

- **Protons** are positively charged particles that reside in the nucleus of the atom. The number of protons in an atom is called the **atomic number**. The atomic number is the bold number written above the element symbol on the periodic chart, and it determines the element (e.g., carbon has 6 protons, argon has 18, uranium has 92). Protons have mass, and make up part of the mass of the atom. Each proton has a mass equal to 1 **amu** (atomic mass unit)

- **Electrons** are negatively charged particles that orbit the nucleus of the atom. The number of electrons in a stable atom equals the number of protons (so there is no net charge). Electrons have no mass (actually, they have an extremely small mass that is so small it is negligible).

- **Neutrons** are particles with mass, but no charge, which reside in the nucleus with the protons. The number of neutrons usually equals the number of protons, however, the number can vary (usually a few more than the protons). The number of neutrons does not affect the properties of the element, but it does affect the mass of the element.

![Figure 14-1. General structure of a carbon atom (© Uliana 2002, 2012).](image)

Elements with the same number of protons, but different numbers of neutrons, are called **isotopes** of the element.
- For example, a stable carbon atom has 6 protons and 6 electrons. Carbon atoms in nature usually have 6 neutrons (6 neutrons +6 protons = carbon 12, or $^{12}$C). Some carbon atoms have 7 neutrons ($^{13}$C) or 8 neutrons ($^{14}$C). These are all isotopes of carbon.
- Each of these isotopes is still carbon; they are all the same size and react with other elements in the same way. The main difference is that $^{13}$C and $^{14}$C are heavier, and $^{14}$C is radioactive (i.e., it is unstable and decays into $^{14}$N).

The mass of the protons and neutrons add up to make the atomic mass of the element (the smaller number written below the element symbol).

**Molecules** contain 2 or more atoms bonded together into a single particle. Molecules can be made up of a single type of element (e.g., nitrogen in the atmosphere is $N_2$), or it could be several elements (e.g., my favorite molecule, $C_8H_{10}N_4O_2$).

**Ions** are elements that have gained or lost 1 or more electrons after they dissolved into water, and so have a positive or negative charge. For example, when table salt (NaCl) dissolves in water, it dissociates into sodium ions ($Na^+$) and chloride ions ($Cl^-$). Molecules that dissociate into ions in solution are called electrolytes.

- Positively charged ions are called **cations**. Examples include
  - calcium - $Ca^{2+}$
  - sodium - $Na^+$
- Negatively charged ions are called **anions**. Examples include
  - chloride - $Cl^-$
  - bicarbonate - $HCO_3^-$
  - sulfate - $SO_4^{2-}$

Note that some ions, like bicarbonate and sulfate, consist of two or more elements.

Some compounds do not dissociate into ions in water; rather they stay in molecular (or nonionic) form. Most organic molecules are nonionic, although some will dissociate into organic cations and anions.

The basic types of chemical bonds (the forces that hold atoms together in a molecule) are:

- **Ionic bonds** – positive ions stick to negative ions (e.g., table salt)
- **Covalent bonds** – two elements share an electron (e.g., many organic molecules)
- **Metallic bonds** – nuclei are arranged in crystals within a “soup” of electrons that are freely shared by all nuclei

**Atomic Mass and the Mole**

As previously stated, the atomic mass is related to the number of protons and neutrons in an element. The mass changes when different isotopes are present, but the average mass of all the isotopes is given on the periodic chart and is sufficiently accurate for use in our calculations.

In any sort of chemistry, we are interested in knowing the number of atoms, ions, or molecules that are being exchanged when reactions occur. For example, consider the reaction of calcite dissolution and precipitation:

$$CaCO_3 + CO_2 + H_2O \rightarrow Ca^{2+} + 2HCO_3^-$$

Each molecule of calcite reacts with one carbon dioxide molecule and one water molecule to produce one calcium ion and two bicarbonate ions. So, if we know how many calcite molecules we have, we can figure out how much water and carbon dioxide we need to dissolve all of it, and how much calcium and bicarbonate will be produced.

However, we don’t measure the number of atoms or ions; what we do measure is mass of the elements. This is a problem because each element has a unique atomic mass. Fortunately, there is a fairly simple way of expressing the number of atoms and relating that to the mass of the elements involved – the mole.

A mole is a number of atoms (as defined by **Avogadro’s Number**, which is $6.02 \times 10^{23}$) in a mass of element equal to the atomic mass in grams. For example:
Oxygen has an atomic mass of 16 (actually, it’s 15.9994, but 16 is close enough for us). Therefore, 16 grams of oxygen contains \(6.02 \times 10^{23}\) oxygen atoms, and 1 mole of oxygen weighs 16 grams.

We can also express compounds as moles by determining the formula weight of the molecule. This is done by summing up the masses of the individual atoms in the molecule. For example:

- The formula mass of \(C_8H_{10}N_4O_2\) is
  - Atomic mass of C = 12 x 8 = 96 amu
  - Atomic mass of H = 1 x 10 = 10 amu
  - Atomic mass of N = 14 x 4 = 56 amu
  - Atomic mass of O = 16 x 2 = 32 amu
  - Total atomic mass = \textbf{194 amu}

Therefore, one mole of caffeine has a mass of 194 grams.

In any given chemical reaction, a certain number of atoms of one element will react with a certain number of atoms of another element. We can use the atomic mass to convert the masses of our elements to moles, then calculate how many moles of each element react with the other elements, then convert back to mass.

Concentrations of Solutions

An element that dissolves into water (or some other liquid) is called a solute. Frequently, the individual elements or molecules that are dissolved will be called dissolved species or dissolved constituents. The liquid it dissolves into is called a solvent. We can measure and express the concentration of the solute in the solvent in terms of the mass of the solute or the number of moles of the solute.

- Mass solute per volume of solution. Most often expressed as mg/L or µg/L.
- Mass of solute per mass of solution. Used to be expressed as parts per million (ppm) or parts per billion (ppb), now they are usually reported as mg/kg or µg/kg.
- Molarity, or molar concentration (M) refers to the number of moles of a species per liter of solution. Molality, or molal concentration, (m) refers to the number of moles of a species per kilogram of solution. These two values are generally the same for aqueous solutions, since a liter of water has a mass of about 1 kilogram. However, the mass of a liter of solution increases as the concentration increases, so for something like seawater, molarity and molality are not the same.

We can use the formula weight of the element to convert between mass concentrations and mole concentrations. To do so, express the concentration as grams per volume, then divide by the formula weight to get moles per volume.

The reason we use mole concentrations is because our chemical reactions involve numbers of atoms rather than masses of atoms. But, in dealing with ionic reactions (like the precipitation or dissolution of salts), the number of ions that will react with each other depends on the charge of the ions. An ion with a +2 charge (e.g. calcium – Ca\(^{2+}\)) will react with two ions with a –1 charge (e.g., chloride – Cl\(^-\)). Therefore, it is sometimes useful to express concentrations in terms of equivalent charge concentrations. The equivalent charge concentration (in equivalents (eq) per liter) is equal to the molarity times the charge of the ion. For example:

- Sodium (Na\(^+\)) has a charge of plus one; therefore, 1 M sodium solution = 1 eq/L
- Calcium (Ca\(^{2+}\)) has a charge of plus two; therefore, 1 M calcium solution = 2 eq/L

Equivalent concentrations are usually expressed in milliequivalents per liter (meq/L) or per kilogram (meq/kg). We can convert from mg/L to meq/L by dividing by the formula weight and then multiplying by the charge of the ion.

Water Analysis and Naturally Occurring Dissolved Species

Groundwater taken from any given place contains a wide variety of dissolved constituents. We have a variety of means for analyzing water, both in the field and in the laboratory, which characterize the concentrations of dissolved solutes in the water. Here we will discuss the things that we measure when we analyze water.
Basic Chemical Parameters

When analyzing groundwater, it is common practice to measure temperature, pH, alkalinity, total dissolved solids (TDS), and specific conductance.

- pH is defined as the inverse log of hydrogen ion activity in the water (activity is basically like concentration). For example, a pH of 7 means that the activity (or concentration) of hydrogen in the water is equal to $1 \times 10^{-7}$ mol/L. A pH of 5.6 means that hydrogen activity is equal to $1 \times 10^{-5.6}$ mol/L.

- Alkalinity refers to the ability of the water to neutralize an acid. This is directly related to specific dissolved species in the water; in most groundwater systems, the predominant acid-neutralizing species is the bicarbonate ion. Therefore, alkalinity is usually used as a direct measurement of the concentration of bicarbonate in groundwater. Alkalinity is usually measured in the field, when the samples are taken, because exposure to air can cause some of the bicarbonate in solution to turn into carbon dioxide and leave the water.

- The total dissolved solids represent the total concentration of dissolved constituents in the water; usually measured by evaporating a liter of the sample and measuring the weight of the remaining solids. TDS is measured in mg/L. Shallow groundwater (within 200 meters of the surface) generally ranges from 100 mg/L to 10,000 mg/L TDS (seawater is 35,000 mg/L). The standard limit for drinking water is generally considered to be less than 1000 mg/L, with less than 500 preferable (although people can tolerate up to 2000 mg/L).

- Specific conductance is a measure of the ability of the sample to conduct electricity. This acts as an approximation of the TDS, since the electrical conductivity of water is a function of the amount of dissolved material in the water.

In addition to these basic chemical parameters, we also analyze for the concentration of individual constituents in the water. Most of the dissolved constituents in groundwater are inorganic cations and anions derived from the dissolution of minerals. In any natural groundwater sample about 95-99% of the dissolved constituents (by weight) will consist of the following:

- Bicarbonate
- Calcium
- Chloride
- Magnesium
- Silicon
- Sodium
- Sulfate
- Carbonic acid

These are usually referred to as the major constituents or primary constituents, and generally have concentrations greater than 5 mg/L. These constituents come directly from the dissolution of mineral phases in the subsurface, and a set of laboratory analyses will generally test for them. Minor constituents, which generally occur in concentrations ranging from 0.1 –10 mg/L, include:

- Boron
- Carbonate
- Fluoride
- Iron
- Nitrate
- Potassium
- Strontium

In addition to these, there are a host of trace constituents that occur in concentrations less than 0.1 mg/L. Some of these are heavy metals that are toxic in small quantities. These constituents are generally not analyzed for.
If we analyze a water sample for the major and minor constituents, we will identify just about all of the mass dissolved in the sample. There are ways that we can check our analytical results, to make sure that we are measuring correctly.

- If we add up the concentrations of the individual species, as determined by various lab procedures, we can compare that sum to the total dissolved solids concentration as determined by evaporation. If the two are different, then most likely there is a problem with the lab analyses.

- Water is electrically neutral; for every positive ion in solution, there is an equal number of negative charges to offset the charge. We can do a charge balance to see if the positive charges equal the negative. This is done by converting the concentrations to meq/L, then adding up the meq of the cations and subtracting from that the meq of the anions. Ideally, they should be within 5% of each other (owing for analytical uncertainty and the trace elements that were missed in the analysis).

A Little about Chemical Reactions

We will not get into the specific details of any of the reactions that occur in groundwater systems. However, a basic understanding of the nature of these reactions, and how we study them, is important.

Most of the reactions that are affecting groundwater chemistry involve reversible reactions, like precipitation and dissolution of mineral phases and certain oxidation-reduction reactions. Reversible reactions are reactions in which the system reaches a dynamic equilibrium. For example, let's consider a glass of water with a piece of calcite in it. The water will dissolve the calcite until it is saturated and can hold no more water. At that point, the system is at equilibrium. However, the reactions don't just stop; what happens is that calcite is precipitating and dissolving at the same rate, so the concentration in the water doesn't change. This is the dynamic part of the equilibrium – the reactions still occur, but are proceeding in both directions at an equal rate. The characteristics of this equilibrium (e.g., how much is in solution, how much is in the mineral, etc.) are based on the thermodynamics of the system – in other words, these reactions are all about changes in energy in the system, and they are all about the system trying to reach a state of minimum energy. A lot of the chemical models that we use to understand these systems are based on the thermodynamics of equilibrium. These models and approaches to groundwater chemistry ignore reaction rates – in other words, if we assume that a system is at equilibrium, we know nothing about how long it took to get there or how fast the reactions are proceeding.

When we study and try to understand the dynamics of geochemical systems, we often make the assumption that the system is at equilibrium. This is just like a lot of other assumptions that we make – it simplifies things for us and turns a complicated system into something that we can understand, but in reality it is wrong. Natural aqueous systems are rarely at equilibrium; instead, they are in a state of moving towards equilibrium at a rate that is related to the reaction rates. In this case, it is more appropriate to use a kinetic approach to modeling the chemistry. Kinetic models are appropriate for reactions that proceed slowly.

Some reactions in groundwater systems are irreversible; they just go in one direction. Examples include radioactive decay, some oxidation-reduction reactions, and degradation of organic contaminants. When dealing with irreversible reactions, the concept of equilibrium does not apply and we are required to use a kinetic approach.

What Do We Do With the Data?

When we collect groundwater samples and analyze them, we compile information about the dissolved species in the water. The dissolved constituents tell us something about the rocks and sediments through which the water flowed, and can be useful to hydrogeologists in their quest to understand flow systems. If we know something about the type of rocks or sediments in the subsurface, and have data on the concentrations of ions in groundwater, we can make some inferences about where the water flowed and what rocks it encountered along the way. Some examples:

- Suppose we have a groundwater sample taken from a well that has a lot of calcium and sulfate in it, and we know that nearby beds of gypsum and anhydrite (CaSO₄·2H₂O and CaSO₄, respectively) are dipping into the subsurface, we can infer that the groundwater must be flowing through the gypsum and anhydrite beds.
Suppose we have two samples, one downgradient of the other, and both samples have very similar chemistry with the exception that the upgradient sample has a lot of calcium and magnesium in it while the downgradient sample has less Ca\(^{2+}\) and Mg\(^{2+}\) and much more sodium (Na\(^+\)). If we know that there are a lot of clays in the area, we may infer that cation exchange is occurring (i.e., the clays are grabbing the divalent cations and releasing monovalent cations).

We will now investigate some of the techniques (graphical and otherwise) that we use for presenting and interpreting groundwater chemistry data.

**Piper Diagrams**

Piper diagrams are a specific type of ternary diagrams used to plot the concentrations of the major cations and anions (Figure 14-2).

The Piper diagram consists of two ternary diagrams (one for cations and one for anions) and a central diamond-shaped field into which the cation and anion values are projected. The relative percentages of the concentrations (in meq/L) of the three major cations (Ca, Mg, and Na) are plotted on the ternary diagram to the left. Usually, potassium and sodium are added together, since they are so similar in size and readily substitute for each other in minerals. The same is done with the three major anions (Cl, HCO\(_3\), and SO\(_4\)). The location of each point is projected up into the central area, and the intersection of the projections defines a point on the central field.

Piper diagrams are good for looking at lots of data, because they can show distinct groups of water types as well as the evolutionary paths that waters take as they move through the subsurface.

An example of actual data on a piper diagram is given in figure 15-3. All data are from an investigation of saline groundwater contamination at a site in Culberson County, Texas. The red symbols are samples from shallow wells that represent the natural groundwater conditions, and the blue symbols are from brine injection disposal wells. Samples from the site fall into two groups; Group 1 are similar to the natural groundwater at the site, while Group 2 is more similar to the brine wells, indicating that they are contaminated with brine.
Stiff diagrams

Stiff diagrams are a way of representing analyses from an individual water sample. A Stiff diagram is basically a graph that has concentration (in meq/L) on the horizontal axis and the major cations and anions on the vertical axis (Figure 14-4). The central vertical line represents zero concentration, and the cation concentrations are to the left while the anion concentrations are to the right. The concentrations are plotted, and the points are joined with a solid line to create a small polygon.

The utility of these diagrams are that they give the viewer an instantaneous picture of the water chemistry. For example, the diagram on the top is from a water sample that contains predominantly calcium and sulfate, while the bottom one is from a water sample that is mostly sodium and chloride. We can use these diagrams to compare samples and to identify groups of samples.

The following is an example of an application of stiff diagrams in an area that was allegedly impacted by saline groundwater released by faulty injection wells.

Several shallow wells located on a ranch in West Texas had suddenly become too salty for the cattle to tolerate. The well owners suspected that brine injection wells (operated by an oil company) located on the ranch property were responsible for the contamination. Samples were taken from a number of wells in the vicinity of the brine injection wells. Figure 14-5 shows the results of those samples, plotted as Stiff diagrams.

The TDS, sodium concentrations, chloride concentrations, and the shapes of the Stiff diagrams suggested that the samples fell into two distinct groups; a lower TDS group with a chemical signature that had more calcium and sulfate, and a higher TDS group that had more sodium and chloride (Figure 14-6). Groundwater samples taken from property off of the ranch suggested that the natural groundwater in this region is dominated by calcium and sulfate, and generally has little sodium and chloride. It was therefore hypothesized, based on these data, that the group 1 samples are more like the natural water chemistry with minimal impact from the brine wells, while the group 2 samples have been heavily impacted by saline water.
When the Stiff diagrams are plotted on a map (Figure 14-7), we see a pattern emerge. The group 2 samples (1, 3, 6, and 8) are closest to the brine injection wells (2 and 7), while the others are farther away. This suggests that the group 2 samples have been contaminated by the brine injection wells, and that the group 1 samples are closer to the natural water chemistry.

Schoeller diagrams are another graphical way of expressing the chemistry of a single sample (Figure 14-8). In these diagrams, the horizontal axis is each of the major cations and anions, while the vertical axis is the concentrations of each in meq/L on a log scale. When the data are plotted, they are connected with straight-line segments to create a crooked line that, like in the Stiff diagrams, gives an immediate picture of the chemistry of the samples that can then be compared to other samples.

Geochemical Maps and Hydrochemical Facies

Geochemical data can be plotted on a map and contoured, just like any data. For example, we could plot total dissolved solids or chloride concentrations on a map at the sample locations and then draw contours to show the distribution of these values. This procedure works well for delineating contamination plumes, because there is a single area where the concentration is largest, and there is a range of values as the contamination is dispersed downgradient. We showed contamination plumes in the last chapter on Mass Transport.

In natural systems, there is generally a great deal of variability from one sample to the next, especially in areas where a lot of recharge (i.e., fresh water) is entering the aquifer. The concentrations of individual ions can change, as well as the overall concentrations of the water. There may be so much variability that contouring the data is really not possible.
However, even though we can’t characterize the groundwater based on the absolute concentrations, we can look at the relative concentrations to see which ions are predominant in each sample. We can then use these to develop hydrochemical facies or waters grouped by the predominant ions in the samples.

Figures 14-9 and 14-10 show how stiff diagrams and hydrochemical facies can be used to characterize the groundwater chemistry throughout a region in West Texas. These maps also suggest how the chemistry may be related to the mineral phases in the subsurface.
Chapter 15 – Surface Water

Introduction

This course deals with groundwater, and to date the majority of the material has dealt with the physics, mathematics, and chemistry of groundwater flow. However, groundwater is directly linked to the surface water part of the hydrologic cycle, and the freshwater runoff system (including evaporation, precipitation, infiltration, runoff, and accumulation in lakes) is the primary avenue of recharge to and discharge from the groundwater. In addition to the links between groundwater and surface water, the surface runoff network (Figure 15-1) is a primary source of water for people, agriculture, and industry, and it is important to have a basic understanding of the system and how we study it. This chapter will deal with surface water and the interactions between surface water and groundwater. 

Figure 15-1. The hydrologic cycle (from US Geological Survey Outreach Website at http://wa.water.usgs.gov/outreach.html)

Basics of Surface Water

When rain hits the Earth, some of it soaks into the ground, some is taken up by plants and animals, and some evaporates into the air. The rest of the water - the part that drains to the nearest creek or river and flows towards the ocean - makes up the part of the hydrologic cycle that we call surface water.

A stream is the general term that we use for any body of flowing water on the Earth's surface that is contained within a channel. Streams receive their water from a surrounding area called a drainage basin or a watershed. Watersheds are defined by the topography surrounding the stream, and watersheds are determined by arbitrarily selecting a point on a stream and delineating the area that drains to that point (Figures 15-2 and 15-3).

Figure 15-2. Schematic diagram of a watershed (© Uliana, 2001).
Some definitions:
- The topographic high that separates one watershed from the next is called a **drainage divide**.
- The place where the stream begins, up in the watershed, is called the **headwaters** or the **end** of the stream.
- The point where the stream finally flows into the ocean (or a lake, or another river) is called the **mouth** of the stream.
- The primary channel in the stream is called the **mainstem**; smaller streams that flow into the mainstem are called **tributaries**.

### Evaporation, Transpiration, and Precipitation

**Evaporation**

Where liquid water exists at the Earth’s surface, water molecules are continually exchanged between liquid and the atmosphere. **Evaporation** occurs when the number of water molecules passing to the vapor state exceeds the number joining the liquid state. The rate of evaporation depends upon the water temperature and the temperature and humidity of the air above the water. **Humidity** refers to the amount of moisture in the air; more specifically:

- **Absolute humidity** – mass of water per unit volume of air (usually grams water per cubic meter of air)
- **Saturation humidity** – maximum amount of moisture the air can hold at a given temperature
- **Relative humidity** – the absolute humidity over the saturation humidity (i.e., the percent ratio of the amount of moisture in the air to the total amount it could possibly hold)

Evaporation from lakes and rivers, and even directly from the groundwater, is a significant flux in the water cycle and must be considered in water-budget studies. Evaporation rates from a lake or a reservoir can be determined indirectly by measuring the inflows, outflows, and changes in storage in the lake, and using the hydrologic equation to fill in the evaporation part. This can be difficult, because it is hard to measure how much water is entering or leaving the groundwater.
Evaporation can be measured directly using shallow pans of water. Pans are maintained throughout the US by the National Weather Service for constant monitoring of evaporation rates. Water is maintained in these pans at a fairly constant depth, and the amount of water added to the pan (by the operators as well as by precipitation) is used to calculate evaporation rates.

One of the complications of using pans to estimate the amount of water that would evaporate from a nearby reservoir is that the pan absorbs heat from the sun much more readily than the reservoir, and the pan can also lose heat (through the sides and bottom) much more easily than the reservoir. Therefore, the pans always overestimate the rate of evaporation. Pan evaporation rates must be multiplied by a pan coefficient, which is a number less than 1 (usually somewhere between 0.5 and 0.8).

**Transpiration**

Plants are constantly pumping water from the ground into the atmosphere through a process called transpiration. Plants take up water for their own use (i.e., for building plant tissue), but only about 1% of what they suck up gets used; the rest is released to the atmosphere through leaves. Transpiration is a difficult thing to quantify; it varies with the time of the day (most during daylight hours, when photosynthesis is occurring) and time of year, and individual types of plants will take up water at different rates.

Transpiration is significant anywhere there are plants, but in some cases it can drastically reduce the amount of water in streams. Plants called phreatophytes extend their roots down into the saturated zone and pump water out at drastic rates. A phreatophyte called *Tamarix*, or ‘salt cedar’, has spread along many river miles throughout the arid southwest, and has significantly reduced flow rates in the rivers.

Other types of plants are xerophytes, which are shallow rooted plants that live in desert areas and require little water, and hydrophytes, which are aquatic plants that live directly in water.

**Evapotranspiration**

When studying water in the field, we cannot separate water lost to evaporation from transpiration losses; therefore we usually lump them together as evapotranspiration (E-T). To understand this we need to distinguish between potential evapotranspiration and actual evapotranspiration.

Potential evapotranspiration is the water loss that would occur if there is an unlimited supply of water available for transpiration and evaporation.

In reality, the amount of water that transpires or evaporates is limited by the amount of water that is available. If the amount of water available is less than the potential, then the actual evapotranspiration will be lower than the potential.

Figure 15-4 shows the relationship between precipitation and potential/actual E-T in an area with a warm, dry summer and a cool, wet fall/winter/spring. In the summer months, when precipitation is low, there is not enough water to satisfy the potential; therefore the actual E-T is less than the potential.
Actual E-T cannot exceed the potential, but if precipitation and the capacity of the soil to store water are both low, then actual can be much less than potential. In areas where precipitation is more evenly distributed throughout the year, actual E-T will be close to potential E-T. This is important because we can measure potential evaporation (i.e. pan evaporation) and determine potential transpiration for specific plants, but we have to take into account that the potential will not be reached if there is not enough water available.

**Precipitation**

As masses of air cool, the saturation humidity decreases, and the relative humidity, in turn, increases. When the relative humidity approaches 100% condensation begins to occur. Condensation forms on particles of dust or ice crystals suspended in the air to form clouds. When air masses rise, they expand, and as they expand they cool and water condenses. If enough water condenses, it will exceed the air’s capacity to hold it and water will fall out as precipitation.

The amount of rain that falls during a storm is usually measured in terms of a depth. This depth refers to the depth of water that is left in a rain gauge after the storm. If we know the depth at any given point, and the area of our watershed, we can determine the volume of water that fell in the drainage basin during the storm. However, the rate and the total depth of precipitation will vary from place to place in an area during a storm. When attempting to determine the amount of rain that falls in a watershed, we need to calculate some sort of average precipitation, or **effective uniform depth (EUD)** of precipitation for the watershed.

If our rain gauge stations are distributed evenly throughout the area, each gauge will represent an equal area of the watershed. In that case, we can take a simple average of all the rainfall values at each station will give you an effective uniform depth. Unfortunately, rain gauges are almost never evenly distributed throughout the watershed, and we will need to adjust the values so that an EUD can be determined. There are two basic ways of doing this.

1. **Isohyet method**: Isohyetal lines are lines of equal rainfall. These are drawn on a map of the watershed just like any contour map. The area bounded by adjacent isohyets is measured, and the average depth of the two isohyets is applied to that area. The areas are then weighted and averaged based on the relative size of each isohyetal area. Figure 15-5 shows an isohyetal map.

   ![Figure 15-5. a) Watershed map with rainfall data from several gauges. b) Isohyetal lines drawn on the map](image)

   The drawback of this method is that, since the distribution of rain is never the same, a new map must be redrawn with each rain event.

2. **Thiessen method**: This is similar to the previous method, except that it weights the areas around each gauge. Adjacent stations in the watershed are connected with a network of lines. A perpendicular line is drawn at the midpoint between each line. These are then extended until they intersect with the nearest line, to create a series of polygons covering the area. The area of each polygon is measured, and the percentage of the total area is determined. The rainfall depths at each station are then...
weighted based on the area of the polygon, and these are averaged to determine the EUD. Figure 15-6 shows a Thiessen polygon map.

![Thiessen polygon map](image)

Figure 15-6. Development of a Thiessen polygon map of the rain gauge network in figure 15-5a. a) Lines are drawn connecting the gauge location. b) A perpendicular line is drawn at the midpoint of each line. c) The perpendicular lines are connected to form the polygons (from Fetter, 2001).

The advantage of the Thiessen polygon method is that you only need to calculate the weighting factors for the gauge stations once.

These methods have limitations in mountainous areas, where orographic effects can create vastly different microclimates over short distances. In such areas, detailed studies of vegetation or more detailed rain gauge coverage is needed to properly distribute the rainfall values.

**What Happens When it Rains?**

When precipitation falls from the sky, several things can happen to it. Some will be intercepted by plants before it reaches the ground; this water can evaporate or can eventually fall to the ground or move down the stems of plants or trees to the ground. Some of the rainfall that reaches the ground can infiltrate into the soil.

![Infiltration capacity](image)

Figure 15-7. Infiltration capacity versus time (from Fetter, 2001).

The infiltration capacity of the soil refers to the rate at which the soil can absorb water. For dry soils, the infiltration capacity is high due to capillary action pulling the water into the soil. As the soil becomes wetter, the infiltration capacity diminishes, and less water can infiltrate.

Infiltration capacity changes as a function of time throughout a rain event and follows an exponential decay curve (Figure 15-7). The horizontal line on the curve ($f_e$) is the equilibrium infiltration capacity, and represents the lowest point that the infiltration capacity will reach.
If the precipitation rate does not exceed the equilibrium infiltration capacity (Figure 15-8A), all the precipitation reaching the land surface will infiltrate and there will be no runoff. If the precipitation rate is greater than equilibrium infiltration capacity, but less than the initial capacity ($f_0$) (Figure 15-8B), initial infiltration will accept all of the precipitation followed by runoff or formation of puddles of water on the surface (called depression storage). If the precipitation rate exceeds the equilibrium infiltration capacity and the initial infiltration capacity (Figure 15-8C), runoff or depression storage will commence immediately.

![Figure 15-8. Relationship between infiltration capacity and precipitation rate (from Fetter, 2001).](image)

Water that infiltrates into the subsurface will percolate vertically unless it encounters the water table or some variation in permeability that causes it to move laterally. Layers of low-permeability material in the subsurface can slow down the vertical percolation of groundwater and cause it to move horizontally towards a stream. This flow of water is called interflow.

**Streams**

Streams are generally hydraulically connected to the underlying groundwater. When groundwater is discharging into the stream, we call the stream a gaining stream. The amount of water that flows into the stream from the groundwater is called baseflow. If infiltration causes the water table to rise, the hydraulic gradient in the groundwater will increase and the amount of baseflow will also increase. For many streams, baseflow is the source of water to the stream except during storms, when precipitation in the watershed exceeds the infiltration capacity and the depression storage is filled. In this case, runoff or overland flow will occur, and this runoff will flow into the stream. Natural runoff is usually a small component of the total volume of water flowing through a watershed; only in arid regions where the streams are losing streams and are not receiving baseflow, is natural runoff a significant contributor. In heavily urbanized areas, with a lot of impervious cover (parking lots, roads, buildings), runoff is a much more significant contribution to the surface water system.

**Discharge**

Discharge is defined as the volume of water moving past a point on a stream in a given period of time. In the simplest terms, stream discharge ($Q$) is equal to the velocity of the water ($v$) times the cross sectional area of the stream ($A$), or $Q = vA$.

![Figure 15-9. Cross-section of stream showing velocity profile (© Uliana, 2001, 2012).](image)

The complication is that water in the stream is flowing fastest in the middle of the stream and is slowest at the edges and along the bed of the stream. Figure 15-9A shows a cross section of a stream flowing out of the page. Figure 15-9B shows contours indicating the changes in velocity (in units of, say, cm/sec) throughout the stream. The line a-a’ shows the velocity profile – the arrows represent velocity vectors.
To measure discharge, we have to determine the average velocity of the water in the stream. We can measure velocity using a current meter. Experience has shown that, in water greater than 2.5 feet deep, velocity measured at 0.2 and 0.8 of the total depth and averaged together will give a good average for the water velocity. If the stream is less than 2.5 feet deep, one measurement at 0.6 of total depth will give an average velocity (Figure 15-10). Measurements are made at regular intervals (e.g., 0.5 foot) across the stream. The velocity is then multiplied by the interval and the depth of the stream in that interval to get a discharge for that 0.5 foot-wide strip of the stream. These individual discharge measurements are added up to get the overall discharge of the stream.

![Discharge measurements using current meter](image)

**Figure 15-10. Cross section of stream showing stream gaging stations (© Uliana, 2001, 2012).**

If a lot of stream gage measurements are made at different times of the year under different flow conditions (i.e., low flow, high flow), a relationship between stream stage (i.e., water surface elevation) and discharge can be developed. This is done by measuring the stage height each time the discharge is measured until you have a lot of measurements, then plotting the values on a graph and connecting them with a curve. This curve is called a stage-discharge rating curve. The utility of this is that it is much easier to measure river stage, so once a stage-discharge curve is developed, you can use it to convert stage to discharge.

![Stage-discharge rating curve](image)

**Figure 15-11. Stage-discharge rating curve (©Uliana, 2003, 2012)**

Stage-discharge rating curves are not linear (Figure 15-11). This is because the cross-sectional area in the stream increases a lot faster than the elevation does. So, as the water level gets higher, the overall area available for water to flow through increases much more dramatically and the discharge will therefore increase faster.

**Thought Question**: What might cause the stream-discharge rating curve for a specific stream to change?

We can also measure stream discharge in small streams using a weir. A weir is basically a dam with a small opening of a pre-determined size and geometry that allows water to flow through. The most common shapes for weirs are the rectangular weir, trapezoidal (or Cipolletti) weir, and the 90° V-notch weir (Figure 15-12).
The equations for the rectangular and V-notch weirs are:

**Rectangular weir**

\[
Q = \frac{3}{\pi} (L - 0.2H)H^{3/2}
\]

(Equation 15-1)

**90° V-notch weir**

\[
Q = 2.5H^{3/2}
\]

(Equation 15-2)

where:

- \( Q \) = discharge in cfs
- \( L \) = length of the weir crest in ft
- \( H \) = head of the backwater above the weir crest in ft

These equations are empirically derived, so the numbers don’t have any real meaning – they are simply coefficients relating water elevation to discharge.

**Manning Equation**

The average velocity of flow in an open channel can be calculated using the Manning equation:

\[
v = \frac{1}{n} R^{2/3} S^{1/2}
\]

(Equation 15-3)

where:

- \( v \) = velocity [L/t]
- \( R \) = hydraulic radius, or the ratio of the cross-sectional area of flow to the wetted perimeter [L]
- \( S \) = energy gradient (or the slope of the water surface) [-]
- \( n \) = Manning roughness coefficient
The wetted perimeter refers to the length of the channel that is in contact with the water. The Manning roughness coefficient is an empirically-derived number that basically describes how the roughness of the channel creates friction that slows down the water in the channel.

This equation is simply a way of determining flow velocity, or indirectly discharge. It can be used to model streams that don’t exist; for example, if someone wants to put a drainage ditch on their property, they can use the equation to estimate how fast the water will flow in the ditch. Or, if there is a storm and water velocities are changing too fast to measure discharge, and you know something about the slope and dimensions of the channel, you can use this equation to calculate flow.

**Stream Hydrograph**

A *stream hydrograph* is a graph that represents river discharge at a single point on a river as a function of time. A stream hydrograph shows how discharge in the stream changes with time and, if plotted with precipitation, also shows how discharge responds to storms. A specific hydrograph for a storm event is called a *storm hydrograph*. Figure 15-13 shows a typical storm hydrograph.

![Stream Hydrograph Diagram](image)

*Figure 15-13. Hypothetical storm hydrograph showing response to a 4-day storm event (© Uliana, 2001, 2012).*

The columns represent daily precipitation. The *flood peak*, or point where flow is highest, occurs at some time after the precipitation peak. This difference in time is called the *lag time*. The lag time is due to the fact that it takes time for water to flow overland or through the subsurface as interflow. The lag time is different for different watersheds, and is a function of the size and geometry of the watershed as well as the geologic materials in the watershed. The part of the curve that is increasing is the *rising limb* of the curve; the part that is decreasing as storm water drains out of the watershed is the *falling limb*.

The hydrograph can be broken up into specific components (Figure 15-14). The flow in the stream is a combination of baseflow from the groundwater and surface runoff from the storm event. The baseflow increases throughout the storm event in response to higher water levels in the groundwater around the stream. At the point where the runoff from the watershed is exhausted and the stream goes back to being only baseflow, the stream is in *recession*.
Thought Question: note that the lag times between the runoff peak and the baseflow peak are different. Why is that?

In the absence of precipitation, the recession part of the hydrograph continues to follow a decreasing curve similar to an exponential decay curve. Eventually, as the groundwater continues to drain and groundwater levels drop further, the stream stops receiving baseflow from the groundwater, and stream discharge would be zero. The shape and slope of the baseflow recession hydrograph is a function of the geometry of the basin as well as the geology and soils characteristics, and is unique to each stream.
Chapter 16 – Unsaturated Zone

Introduction

We’ve looked at the groundwater, and we’ve looked at the surface water, now let’s look at what is between the two. There is a layer in the subsurface, commonly referred to as the unsaturated zone, which contains some amount of moisture below the level of saturation. This part of the subsurface is important for a number of reasons – the soils and moisture in this zone support plants, chemical and organic processes filter contaminants and affect the chemistry of groundwater, and the unsaturated zone is an avenue of recharge to unconfined aquifers. The same basic laws of physics that govern the flow of fluids in saturated units are at work in the unsaturated zone, however, the actual processes involved are much more complicated in the unsaturated zone than they are in saturated rock and sediment. In this chapter, we will discuss some of the processes at work in the unsaturated zone.

Distribution of Water in the Shallow Subsurface

Saturation

At the surface of the Earth, the soil, sediment, or rock will have some porosity, and some fraction of that porosity will contain water. This fraction of porosity that contains water is called the degree of saturation ($S_d$), and is defined as:

$$S_d = \frac{\text{volume}_{\text{water}}}{\text{volume}_{\text{voids}}}$$  

(Equation 16-1)

This quantity can be expressed as either a fraction or, if multiplied by 100, as a percentage.

The degree of saturation tends to increase with depth, and the point where the degree of saturation reaches 100% marks the boundary between the saturated zone below and the unsaturated zone above (Figure 16-1).

![Figure 16-1. Cross-section showing the distribution of water in the shallow subsurface (© Uliana, 2001, 2012).](image-url)
This boundary is often considered to be the same as the **water table**, which we previously defined as:

- the point in an unconfined aquifer where the **pore-water pressure** and the atmospheric pressure are equal, and
- the level to which water will rise in a well completed in an unconfined aquifer.

It may seem like we are making this unnecessarily complicated, and we should just define the water table as the boundary between the saturated and unsaturated zones, but the truth is that the two are not exactly the same because of a well-known process called **capillarity**.

**Capillary Forces**

To understand the role of capillary forces in groundwater, we first need to understand a little about pore fluid pressures.

![Figure 16-2. Coffee cup half full with water (© Uliana, 2001).](image)

Let us consider a coffee cup that is half full of water (or half empty depending on your emotional outlook at the moment). Atmospheric pressure is pushing down on the water in the cup (Figure 16-2), and at the surface of the water the fluid pressure in the water is equal to the atmospheric pressure (for every force there is an equal and opposite force, as you recall from your high-school physics class). Then below the surface, fluid pressures increase with depth in the cup as the column of water above increases.

Now let us consider that the coffee cup is full of sand. If we pour some water into the cup so that the bottom half of the cup is saturated, we will have a surface in the middle of the cup that separates the saturated zone from the unsaturated zone. Atmospheric pressure is pushing down on the water in the pore spaces in the sand just like in the cup in figure 16-2, and at the surface of the sand, the **pore fluid pressure** is equal to the atmospheric pressure (Figure 16-3a). At any point below the surface, the pore fluid pressure is equal to the atmospheric pressure plus the pressure associated with the column of water above that point.

Note: Up to this point, we have ignored atmospheric pressure in our calculations of hydraulic head in groundwater. This is because atmospheric pressure becomes negligible a short depth from the water table, and we do not need to consider it when dealing with flow below the water table. When dealing with unsaturated zone problems, however, pore fluid pressures are less than atmospheric, and it will need to be considered.

![Figure 16-3. Schematic diagram showing the interface between the saturated zone and the unsaturated zone in a) an unconfined aquifer with no capillarity, and b) an unconfined aquifer with capillarity (© Uliana, 2001, 2012).](image)
The polarity of water molecules gives water a high surface tension and makes it want to stick to things. The force associated with surface tension is enough to counteract the force of gravity and pull water upwards. This can be demonstrated by dipping a paper towel into water and watching the water move up into the towel against the pull of gravity. Porous rock or sediments are just like the paper towel – there is a lot of surface area in the pore spaces, surface tension causes water to stick to those surfaces, and water gets pulled upward from the water table. This force pulling water up into the pore spaces above the water table is called capillarity, or the capillary force. The capillary forces at work in the unsaturated zone pulls enough water up above the water table to create a thin saturated zone above the water table called a capillary fringe (Figure 16-1).

The capillary forces that are pulling water up into the capillary fringe are offsetting the forces of gravity and atmospheric pressure that are pushing water down towards the water table. As we move from the water up into the capillary fringe, pore fluid pressures decrease. We can then define the capillary fringe as the part of the saturated zone in which pore fluid pressures are less than atmospheric pressures.

Figure 16-1 shows that we separate the subsurface into the saturated zone and the unsaturated zone, and the surface separating the two is the top of the capillary fringe. We also separate it into the vadose zone – the zone where pore fluid pressures are less than atmospheric pressure – and the phreatic zone – the place where pore fluid pressures are greater than atmospheric pressure. The surface that separates these two zones is the water table.

Often you will hear “vadose zone” and “unsaturated zone” used interchangeably (as well as “phreatic” and “saturated”). This is usually ok, since the capillary fringe is generally thin and probably not that important in most groundwater flow problems. However, be aware of what vadose and phreatic really mean, and that in a strict sense, vadose and unsaturated are not the same.

Thought Question: The thickness of the capillary fringe is not constant; it varies from aquifer to aquifer and from place to place in a single aquifer. What factors affect the thickness of the capillary fringe?

Movement of Water in the Vadose Zone

The ease with which water moves in the vadose zone is directly associated with the moisture content of the rock or sediments. Before we talk about the movement of water in the vadose zone, let’s have a quick review of the ways we express moisture content.

Moisture Content

We have already reviewed degree of saturation, or the fraction of the pore space that is filled with water. Moisture content can also be expressed as a gravimetric moisture content (i.e., moisture content by weight) or a volumetric moisture content (i.e., moisture content by volume). The gravimetric moisture content ($\omega$) is the weight of water in the sample divided by the weight of the solids in the sample, or:

$$\omega = \frac{\text{weight of water}}{\text{weight of solids}}$$  \hspace{1cm} (Equation 16-2)

Volumetric moisture content ($\theta$) is the volume of water in the sample divided by the total volume of the sample, or:

$$\theta = \frac{\text{volume of water}}{\text{volume of total}}$$  \hspace{1cm} (Equation 16-3)

Note that volumetric moisture content is always some value less than the porosity of the rock.

Pore-Water Tension

In the vadose zone above the capillary fringe, water exists as a film or coating around the sediment or soil grains that is held to the grains by surface tension. This tension can be thought of as a negative pressure head or matric potential, and we commonly use $\psi$ to denote this quantity.
We can use an instrument called a tensiometer to measure the negative pore pressures in the unsaturated zone (Figure 16-4). A tensiometer consists of a sealed tube with a porous ceramic cup at one end and some sort of pressure gauge on the other end. The tube is filled with water and the pressure gage set to zero, so that the pressure in the tube is the same as atmospheric pressure outside of the tube.

The tensiometer is inserted into the ground so that the ceramic cup is in contact with the soil. The negative pore pressures in the soil will try to pull water out of the tensiometer through the cup; these pressures are transmitted to the fluid and show up on the pressure gauge.

The amount of tension, or negative pressure head, is directly related to the moisture content of the sediments.

With low moisture contents (Figure 16-5a), the film of water on the sediments is very thin, the volume of water in the film is small compared to the surface area of the water, and the surface tension is very high. When the tension is high, the negative pressure head is high, and water movement is impeded by the surface tension. When moisture content is higher (Figure 16-5b), surface tension is lower, and water moves more easily.

This suggests that the negative pressure head is related to the moisture content. We can also deduce that the permeability of sediments in the vadose zone is directly related to the moisture content. This is in fact true – increasing moisture content increases the permeability, and decreasing moisture content decreases the permeability. Unfortunately, the relationship is not linear – in fact, there is an extremely complicated relationship between moisture content and flow, and the theory of flow in the unsaturated zone is not as simple as in the saturated zone.

The general relationship between moisture content and these other parameters (matric potential and permeability) can be seen in figure 16-6. Permeability (hydraulic conductivity) increases as moisture content increases, while matric potential decreases with increasing moisture content. Matric potential is a negative number, so decreasing matric potential is like increasing the overall gradient that pushes water through the unsaturated zone.
Figure 16-6. Graphs of volumetric moisture content versus a) hydraulic conductivity and b) matric potential. These graphs are with data from specific experiments, but they show the general trends (© Uliana, 2012).

Theory of Flow in an Unsaturated Porous Medium

Flow in the unsaturated zone is basically vertical and downward – water is pulled down under the force of gravity. In saturated groundwater, flow is driven by the hydraulic gradient, which refers to the distribution of hydraulic heads in the flow system. Flow in the vadose zone is also driven by differences in head, and the head at any point in the unsaturated zone is similar to the saturated zone head in that it composed of an elevation component and a pressure component. More specifically, unsaturated zone head ($h_u$) is defined as:

$$h_u = \psi + z$$

where:

$\psi$ = matric potential (or negative pressure head) [L]

$z$ = elevation head [L]

Darcy’s law is also applicable in the unsaturated zone, since we are dealing with laminar flow, so we can describe the permeability in terms of a hydraulic conductivity. However, as previously stated, the matric potential and the hydraulic conductivity are both a function of the volumetric moisture content ($\theta$). The implication is that, unlike in saturated sediments, hydraulic conductivity is no longer a constant, rather, it will change depending upon $\theta$. When you think about it, the nonlinear nature of unsaturated flow becomes apparent – water infiltrates into the unsaturated zone at a rate that is dependent upon the moisture content, however, as it infiltrates, the moisture content changes, permeability changes, water infiltrates faster, changing permeability more… At the same time, the matric potential is changing as well, so as the water infiltrates, matric potential decreases, the gradient changes, more water infiltrates… So the problem in modeling the unsaturated zone is that both your primary parameters change as conditions in the model change, and the changes are not linear (i.e., doubling $\theta$ does not necessarily double $K$). Not only do they change, but they change independent of each other – the magnitude of change of $K$ will not be the same as for $\psi$ for the same change in $\theta$. This is what we mean when we say “nonlinear”.

But that’s not the worst of it. Not only do these parameters change, but the magnitude of the change depends on whether water is being added to the soil (wetting) or water is being removed from the soil (drying). In other words, the shape of those curves in figure 16-6 will change depending on how the conditions are changing. This process is called hysteresis – rates of change in a physical process will change depending upon the direction of the reaction. Figure 16-7 shows idealized curves of moisture content versus matric potential and hydraulic conductivity versus matric potential with separate curves for the drying path and the wetting path.
Figure 16-7. Idealized plots of a) water content and b) hydraulic conductivity versus matric potential showing separate curves for the drying and wetting paths (© Uliana, 2012).

Because of this, if we measure $\psi$ using a tensiometer and try to use that to calculate $K$, we will need to know something about the prior moisture history of the soil.

**Studying the Unsaturated Zone: Instruments and Techniques**

**Tensiometer** – used to measure matric head (also called soil suction). These can be installed at varying depths to infer moisture profiles. Usually these are installed between 15 cm and 1 meter below the surface.

**Lysimeter** – like a tensiometer, but instead of measuring matric potential, it is used for sampling of the moisture in the pore spaces of the soil. Has a tube and a porous cup, the tube is left empty and a partial vacuum is generated in the tube through a valve at the top. The ceramic cup is placed in the soil, and the vacuum in the tube draws water in from the soil, where it can be sampled via the sampling tube (Figure 16-8).

Figure 16-8. Lysimeter (© Uliana, 2001, 2012).

**Gypsum blocks** – small (1-2 cm scale) blocks of gypsum with wires embedded; the wires are hooked up to a box that measures electrical resistance. These are buried in the soil with the wires exposed. They can be used to indirectly measure soil moisture (gypsum is not very conductive, so moisture decreases resistance).

**Guelph permeameter** – a portable permeameter that measures the saturated conductivity of the soil. It gives a maximum permeability that can be used to estimate infiltration rates at saturation.

A common method for using tensiometers, lysimeters, and gypsum blocks is to set up a field experiment called an **infiltration plot**. In an infiltration plot, a plot of ground is isolated using some 2x12 boards or a
ring of sheet metal to create an enclosed area that can be filled with water to a certain depth. These instruments are installed within the infiltration plot at various depths and monitored for a few days to record the initial conditions. Then the plot is filled with water, and the instruments are monitored over the next few days as the water infiltrates into the ground. From this data you can develop a series of moisture profiles that illustrate movement of the wetting front down through the subsurface (see figure 16-9).

Figure 16-9. Moisture profiles showing the movement of a wetting front from an initial slug of water in an infiltration plot (© Uliana, 2012).
Chapter 17 – Flow in “Non-Porous Media”

Introduction

To date, our treatment of groundwater flow has some implicit assumptions concerning flow in a porous media. Basically, when we say “porous media”, we are dealing with water moving through pores that are evenly distributed and very small, and our conceptual and mathematical understanding of fluid flow is based on the concept of laminar flow driven through this media by a hydraulic gradient where the flow is linearly and directly proportional to the gradient.

However, this is often not the case – aquifers are not always homogeneous layers of sand or carbonate rocks with evenly-distributed pore spaces. Groundwater flow in many geologic units is controlled by discontinuities in the rock or sediment – cracks, bedding planes, breccia zones along faults, and conduits formed by dissolution of the bedrock. The physics of groundwater flow in these discontinuities is different from flow in a porous media system, and it does not take a very large fracture or conduit to dominate the flow system. Flow in these discontinuities, or in “non-porous media”, is the subject of this chapter.

For our purposes, a fracture is defined as any relatively planar discontinuity in a rock or sediment (yes, even unconsolidated sediments can have fractures). We tend to think of fractures as a break in a rock, but for hydrogeologists, a bedding plane acts the same as a stress-release joint in bedrock. When we look at rocks in nature, we find that all rocks exposed at the Earth’s surface are fractured to some degree. In many rocks, the fractures are not a significant part of the flow system, and it is often a valid assumption to ignore them or to assume that flow in them is acting the same as flow in a porous media. However, fractures are frequently a significant part of an aquifer flow system.

Conceptualizing Fractured Media

The term fractured media refers to an aquifer or formation that has fractures that are a significant part of the flow system. Fractured media can be classified into four categories based on hydraulic properties of the fractures relative to the matrix, or blocks of rock between the fractures (Figure 17-1). The hydraulic properties of interest are:

- Permeability (or hydraulic conductivity)
- Volume of fluid storage

Please be aware that these are idealized conceptualization of real-world phenomenon, and that the differences are gradational and real-world systems may be some combination of two or more categories.

In a purely fractured system (Figure 17-1A), the permeability and porosity of the matrix is so low that it is basically impermeable; all fluid flow in the system occurs in the fractures and all fluid stored in the system is stored in the fractures.

Can you give an example of this type of system?

In a fractured formation (Figure 17-1B), the permeability of the matrix is relatively low, but the porosity is significant. In these systems, the permeability of the fractures so much higher that all of the fluid flow is in the fractures, but most of the water is stored in the matrix. As we will see later, it does not take a very large fracture to have a high permeability. Fractured formations are very significant in the petroleum industry; in
many petroleum reservoirs, most of the oil is stored in the matrix, but it has to flow through the fractures to get to the wells.

In the **double porosity** system (Figure 17-1C), permeability of the matrix is not much higher than the fracture, so flow in the matrix becomes significant. As with the fractured formation, most of the fluid is stored in the matrix. These systems are very difficult to model and quantify, because flow in the fracture is much different from flow in the matrix, and at the same time, there is a great deal of interaction between the two (water moving from the matrix into the fracture and vice versa).

The final system is called a **heterogeneous system** (Figure 17-1D). In this system, the fractures have been filled in with sediment that has a different permeability than the matrix – frequently lower. In this case, the fractures may act as barriers to flow, rather than conduits.

Figure 17-1 introduces the concept of a **fracture skin**. The fracture skin refers to a zone along and immediately beneath the surface of the fracture in which the matrix has been altered. This alteration could be from precipitation of minerals (iron or manganese oxides, calcite), dissolution of minerals from the matrix, or deposition of clay particles in pores (from transport through the aquifer or from alteration of feldspars and micas in the matrix). Fracture skins are significant because they affect the hydraulic relationship between the matrix and the fractures.

In most cases, fracture skins have a lower permeability than the matrix, and they act to impede the exchange of water between the matrix and the fracture. In some cases, especially where the fracture skin has experienced dissolution, the skin can have a higher permeability than the matrix. In these cases, it probably does not have much of an effect on fluid flow.

**Fracture Properties**

**Orientation**

Fractures are three-dimensional structures that are generally assumed to be planar or close to planar. Therefore, just like any planar structure we can characterize the orientation with strike and dip, where the **strike** is the line created by the intersection of the fracture with the horizontal, and the **dip** is the angle of the plane below the horizontal. We can measure these orientations in the field or on aerial photos, and then use rose diagrams (Figure 17-2), radial plots, or stereonets to characterize the distribution of these orientation.
Rose diagrams can be plotted on maps to show fracture trends and changes in orientation throughout a region. These diagrams can identify sets of fractures that may have formed. These can then be related to groundwater flow directions using a number of methods, such as geochemical tracers.

**Aperture**

The aperture is the distance between the fracture walls (Figure 17-3).

![Figure 17-3. Fracture aperture and asperities on fracture wall (© Uliana 2002, 2012).](image)

In an idealized fracture with smooth walls and a constant aperture (i.e., the fracture walls are parallel to each other), the hydraulic conductivity and the discharge are proportional to the aperture. However, real fractures always have asperities that stick out from the wall of the fracture. These affect the properties of the fracture in a number of ways:

- Asperities create roughness on the fracture surface. This roughness has several effects:
  - **Channelization** of fluid along the surface of the fracture – water must follow a more tortuous path (Figure 17-4).
  - **Tortuosity** (τ) – defined as the actual length the water travels divided by the straight-line distance between two points – can also be defined as the reciprocal (Figure 17-5).

![Figure 17-4. Channeling of water along surface of a fracture (© Uliana 2002).](image)

![Figure 17-5. Tortuosity (© Uliana, 2001, 2012).](image)

- Friction associated with surface roughness impedes the flow of fluids. Friction also affects the geotechnical properties of the fracture – higher roughness means that the fracture is less likely to fail.
- In the subsurface, where the fracture is under compressive stress, the asperities hold the fracture open.

**Length/Width/Depth**

Fractures are three-dimensional structures that, in addition to aperture, have a length and a width. An idealized fracture in the subsurface, which does not intersect the surface, will generally have an irregular elliptical shape with an aperture that is maximum near the middle and tapers off to the edges (Figure 17-6).
When we talk about length and width, we are generally talking about the dimension that is exposed at the Earth’s surface – either at an outcrop or identified from an aerial photo. In outcrop, we frequently cannot see the ends of the fracture – it either terminates at the top of the outcrop or it terminates somewhere underground (or both). Length is generally used to describe the exposed length of a vertical fracture in an outcrop; width is often used to describe horizontal fractures. Depth refers to the distance that the fracture goes into the outcrop. This can sometimes be measured or observed using geophysical methods (ground penetrating radar, shallow seismic, electromagnetic) or using boring logs.

**Spacing and Density**

Any given volume of rock will not have a single fracture in it; there will be a number of fractures, and some of those fractures will have similar orientations due to the fact that they were formed by the same stresses. The **spacing** of the fractures refers to the distance from one fracture to the next; the **density** of fractures refers to the number of fractures in a unit volume of rock.

- Spacing is measured by running a scanline or traverse across an exposed rock face and measuring the distance between the points where the fractures intersect the scanline.
- Density is measured by adding up the lengths of all the fractures in a volume or area of rock and dividing that by the volume (or area). We frequently use area because these studies are often two-dimensional outcrop studies (i.e., on the surface) rather than three-dimensional subsurface studies.

Spacing and density seem to be related to the aperture. Studies have indicated that, in specific rocks, there are direct relationships between fracture spacing and aperture; i.e., smaller apertures tend to be more closely spaced, and vice versa. This relationship seems to hold across a number of scales (from thin section to outcrop to air photo), and if it holds true it could have a major impact on the way that we model fractured systems.

**Connectivity**

Fractures are only effective as conduits for fluid flow if they are connected to each other. This is especially true in purely fractured or fractured formation systems. Fracture connectivity can be evaluated by observing fracture terminations and plotting the results on ternary diagrams. There are four basic types of fracture terminations (Figure 17-7A):

- **Abutting** (A) – the fracture end in the middle of another fracture
- **Blind** (B) – the fracture ends in the middle of the rock
- **Crossing** (C) – the fracture crosses another fracture
- **Diffuse** (D) – the fracture ends in a diffuse set of smaller fractures

The number of each type of fracture termination can be counted in outcrop and plotted on a ternary diagram. Since there are four categories, and only three end-members on a ternary diagram, two of the categories must
be grouped. There are two different ways of plotting; the Barton and Hsieh method (Figure 17-7b) and the Laubach method (Figure 17-7c).

![Figure 17-7. A) Fracture terminations; B) ternary diagram as proposed by Barton and Hsieh; C) ternary diagram as proposed by Laubach](image)

In the Barton and Hsieh method, blind and diffuse terminations are lumped together. In the Laubach method, the abutting and crossing terminations are lumped together as ‘connected’.

**Thought question:** Figure 17- 8 shows ternary diagrams from two different fractured formation aquifer (one aquifer (Figure 17-8a) is a basalt, the other (Figure 17-8b) is a highly-cemented quartz sandstone). Each dot represents an outcrop; for each aquifer about 9-10 outcrops were studied.

![Figure 17-8. Fracture termination data from two different aquifers (© Uliana, 2001).](image)

Based on these results alone, in which aquifer would you expect to see the higher permeability?

**Formation of Fractures**

There are generally three fracture modes of loading, which involve different fracture surface displacements (Figure 17-9). The three modes are:

- Mode 1: opening or tensile mode (the fracture faces are pulled apart) (Figure 17-9a)
- Mode 2: sliding or in-plane shear (the fracture surfaces slide over each other) (Figure 17-9b)
- Mode 3: tearing or anti-plane shear (the fracture surfaces move parallel to the leading edge of the fracture and relative to each other) (Figure 17-9c)
Different fracture modes will result in different fracture properties. Modes 2 and 3 have shear stresses that will affect the walls of the fracture – asperities will be sheared off and minerals may form on the surface (“slickensides”).

As a fracture forms and propagates through a rock, the stresses that are breaking the rock are distributed throughout the rock so that high compressive stresses occur just ahead of the tip of the fracture. Because of this, when two fractures propagate towards each other, the stresses of each will deflect the direction that the other propagates (Figure 17-10).

Stresses that create fractures include:

- Tectonic stresses
- Unloading and exfoliation - when a rock is buried, it is under compressive stress; when the overburden is released, the release of stress causes the rock to fracture.
- Cooling joints – cooling lava will often contract, creating joint sets that are usually vertical and often result in sets of polygonal columns (“columnar joints”)
- Gravity and freeze-thaw – can cause existing fractures to propagate further
Hydraulic Properties of Fractures

Porosity

If we recall the porosity lab included in the lab packet, we calculated the porosity of a granite with three sets of orthogonal fractures with a constant aperture of 1 mm and a spacing of 1 meter. The result was a very low number (0.3% porosity). This is true in most natural formations; the porosity associated with fractures is very low, and is generally much lower than the matrix porosity (unless you are dealing with a purely fracture system in which the matrix has very little porosity). There just isn’t a whole lot of porosity associated with fractures. Permeability, however, is a different story.

Permeability

Discharge in a single idealized fracture with smooth walls and a constant aperture is governed by an equation called the cubic law. The cubic law is:

\[ Q = \frac{\rho_gb^3i}{12\mu} \]  

(Equation 17-1)

where:

- \( Q \) = volumetric discharge [L³/t]
- \( \rho_w \) = density of water [M/L³]
- \( g \) = gravity [L/t²]
- \( b \) = aperture (distance between walls of fracture) [L]
- \( i \) = hydraulic gradient (dh/dl) [-]
- \( \mu \) = viscosity of water [M/L·t]

Based on this equation, the hydraulic conductivity of a fracture is:

\[ K = \frac{\rho_gb^2}{12\mu} \]  

(Equation 17-2)

What we see from this equation is that hydraulic conductivity is proportional to the square of the aperture, and discharge is proportional to the cube of the aperture (hence the name “cubic law”).

This is the basis of our conceptual and mathematical understanding of fluid flow in fractures:

- In an idealized fracture, the hydraulic conductivity and discharge are a function of, and directly proportional to, the size of the aperture (i.e., an increase in aperture results in an increase in the hydraulic conductivity and an increase in the discharge).
- The relationship between aperture and conductivity/discharge is not linear – a small increase in aperture results in a large increase in permeability and an even larger increase in discharge.

We can use the cubic law to calculate discharge and permeability of fractures, and when we do, we see that a fracture as small as 1 mm has permeability close to 1 m/s and can transmit a lot of water. Fractures don’t have to be very large to have a big influence on a flow system.

Modeling Fluid Flow in Fractures

There are a number of ways we can conceptualize fluid flow in fractures. We won’t get into the mathematics associated with each, but it is important to conceptually understand the methods.

Parallel Plate Models

A purely fractured system can be modeled using the cubic law and the assumptions that 1) all fluid flow is in the fractures, and 2) the fractures can be represented as a series of parallel plates (in other words, the fractures are all smooth-walled and have constant apertures). The spacing and density of fractures in a volume of rock can be used to integrate the cubic law over the whole volume, so that a hydraulic conductivity tensor for the
whole volume is calculated. There are also ways to factor in roughness and variations in aperture; usually these involve multiplying the cubic law by some sort of factor that reduces the flow accordingly.

This method has some limitations. It has an implicit assumption that the fractures follow a constant spacing, or some sort of spacing that varies in a predictable way. Also, it ignores flow in the matrix, which may or may not be a good assumption.

Discrete Fracture Model
If the exact locations, extents, orientations, and hydraulic properties of all the fractures in a system are known with some degree of confidence, the system can be simulated using a model that contains and calculates flow in each fracture. This is used frequently in the petroleum industry, when a reservoir has been extensively studied (through drilling and geophysics) and the locations of the major fractures are known with some certainty. The obvious limitation with this is that, to get a meaningful model, you need to have lots of data. For many groundwater problems, this is not within the budget and may simply not be available.

Stochastic Models
Stochastic basically means random, or statistical. In stochastic models, the probability of occurrence of a fracture is input to the model rather than actual orientations. These can be used where data is sparse, and are most effective when it can be shown that the fractures follow some sort of predictable distribution.

Double-Porosity Models
Some numerical models incorporate a parallel-plate or discrete fracture approach with a standard porous media model approach to simulate flow in a double-porosity system. A difficult part of these models is the relationship between the fractures and the matrix blocks – there has to be some sort of mathematical function that controls the flow of water from fractures to the matrix and vice-versa, and this function is most likely non-linear. There are several computer programs on the market that do just this (e.g., FRAC3DVS).

Equivalent Porous Media
A simple way of modeling double-porosity systems is by basically ignoring the fractures and developing some equivalent hydraulic parameters that characterize the aquifer as a single porous media. The idea is that, even though your system is fractured, the model contains a representative elemental volume (REV) of the formation or aquifer. This is frequently done; probably because the modelers don’t have enough data to do anything else. This method works if the fractures are so numerous and well-connected that the whole aquifer acts as an equivalent porous media, but it does not work well if a few large fractures are controlling the flow system.

Equivalent Porous Media with Fracture Domains
A more appropriate way of using the equivalent porous media approach is by establishing fracture domains, or areas in which the fractures have common characteristics (orientation, connectivity, etc.) (Figure 17-11). Investigations (air photo, outcrop) can be used to establish regions with similar fracture characteristics. The parallel plate approach can be used to establish a hydraulic conductivity tensor for each region. Then these can be input into a model as a heterogeneous porous media model.
Influence of Fractures on Fluid Flow and Mass Transport

Fractures affect fluid flow in a number of ways. The obvious way is that open fractures provide a conduit for water to move in the subsurface.

- Fractures can impart a significant anisotropy in an aquifer; affects flow paths
- Fractures can control the path of contaminants in the subsurface; contaminants may travel in unexpected directions
- Contamination can show up sooner than expected
- Fracture skins – influence on mass transport (impede diffusion into matrix; can also increase sorption)

Fractures can also act as barriers to flow in an aquifer

- Mineral fill in veins or in fault breccias
- Juxtaposition of permeable beds against impermeable
Chapter 18 – Consolidation and Subsidence

Introduction

Earlier, we discussed the response of a confined aquifer to pumping from a well. If you recall, when a well is pumped in a confined aquifer, the pore spaces stay filled with water, and the water comes from the elastic response of the aquifer. What happens is that, as pressure is reduced in the pores, the pores get a little smaller and the overall pore volume decreases slightly. When we discussed this earlier, we did not consider the fact that changing the volume of the individual pores might change the overall volume of the aquifer. Well... it does. As the fluid pressure decreases, the grains expand, the pores get smaller, and the overall volume of the aquifer decreases. This is called consolidation. Just as different rocks react to tectonic stresses in different ways, different aquifers and hydrostratigraphic units react to changes in fluid pressure in different ways. That is the subject of this chapter.

Review of Elastic Response in a Confined Aquifer

Let's consider a confined aquifer that consists of a layer of sand overlain by a layer of clay that acts as a confining layer.

If we stick a well into a confined aquifer and pump water out of it, we begin to lower the heads around the well in such a way that we create a cone of depression in the potentiometric surface (Figure 18-1). The more we pump from the well, the larger the cone of depression gets. Now, if we look at the example in figure 18-1, we see that we have removed a volume of water and have lowered the water level in the well and in the surrounding aquifer. However, the saturated thickness of the aquifer does not change (i.e., the aquifer is still full up to the base of the confining layer).

What is happening has to do with the additional fluid pressure, or excess fluid pressure, in the confined aquifer (Figure 18-2). The pressure head in the aquifer at the well screen is provided by the column of water above that point in the aquifer. In an unconfined aquifer, the pressure head is equal to the thickness of the saturated zone above the well screen. In a confined aquifer, there is the thickness of the saturated zone plus the additional pressure that is reflected in the elevation of the potentiometric surface above the base of the confining layer.
To put it in other words, in a confined aquifer, there is some additional amount of pressure in the pressure head component—this is represented by the length of the column of water in the well above the top of the aquifer. If there was no additional pressure (like in an unconfined aquifer), the water level in the well would rise to the top of the water table. It is the extra pressure in the confined aquifer that is pushing the aquifer matrix apart and creating extra space for the water in the aquifer, and when that pressure is relieved, the grains will expand and relax back to their original positions (Figure 18-3).

Decreasing the fluid pressure in the aquifer causes the potentiometric surface to drop and the individual pore spaces to decrease. The change in volume of each individual pore is miniscule, but can add up to a significant change in volume of the aquifer.

What we are talking about here is a strain (i.e., change in volume or change in length) resulting from a stress (some force or pressure applied in three dimensions). The stresses in an aquifer are due to the weight of the column of rock and water above a given point, and the strains that we observe are consolidation of aquifer materials, which can often result in subsidence of the ground surface. Before we talk about strain (i.e., consolidation), let’s talk about the states of stress in the subsurface

States of Stress in the Earth’s Crust

At all points in the subsurface, the rocks and sediments are under stress. Stresses can be normal or shear relative to the point of observation, and normal stresses can be either compressional or tensional (Figure 18-4).

Stress is a tensor quantity, meaning that it exists in three dimensions in the subsurface and the magnitude of stress varies with respect to location. Fortunately, when dealing with basic hydrogeology and fluid-related consolidation problems, we can ignore the horizontal stresses and only deal with vertical stress. If we are dealing with an area that is at a tectonically active margin, the horizontal stresses may be significant. However, we will ignore those in this chapter, and only consider the vertical stresses. In other words, from now on all we will deal with is stress in the vertical direction (making thing much, much easier).
The vertical stress at a given point in the subsurface is simply a function of the pressure exerted on that point by the weight of the overlying material. This pressure is equal to the weight of the material times the thickness of the material. To put it all together into an equation:

\[ \sigma_v = \rho_b g z \]  

(Equation 18-1)

where:
\( \sigma_v \) = vertical stress [M/L·t²]
\( \rho_b \) = bulk density of the overlying material [M/L³]
\( g \) = gravity [L/t²]
\( z \) = thickness of the layer of material [L]

This equation calculates the total stress at any given depth in the subsurface, based on the thickness of the overlying material. The total stress, also called the **lithostatic stress**, at that point is the sum of two different stresses: the **effective stress**, which is the stress from the column of rock above the point (Figure 18-5), and the **pore fluid pressure**, which is the stress related to the fluid in the pores (Figure 18-3). This relationship is summed up by an equation called **Terzaghi’s Law**, which states:

\[ \sigma = \sigma' + p_f \]  

(Equation 18-2)

where:
\( \sigma \) = total stress; also called the normal stress (if we are only considering vertical stresses)
\( \sigma' \) = effective stress, or “grain to grain” stress in the rock
\( p_f \) = pore fluid pressure

Figure 18-5. a) Close-up of grains in the subsurface; b) same grains with grain contacts highlighted. The effective stress is the stress related to the weight of the sediment grains and is supported by the grain contacts (© Uliana, 2001).

So, we’ve got these two stresses – the effective stress, which is pushing down, and the pore fluid pressure, which is really acting in all directions but is considered to be acting opposite the effective stress.

In an unconfined aquifer, the pore fluid pressure is simply the pressure from the column of water above the point in the aquifer (just like the total stress is the pressure associated with all the material above that point). This is called the **hydrostatic pressure**. We can use equation 18-1 to express the hydrostatic pressure as:

\[ p_h = \rho_w g z \]  

(Equation 18-3)

\( p_h \) = hydrostatic pressure
\( \rho_w \) = density of water
\( z \) = thickness of the overlying layer of water [L]
In a confined aquifer, the pore fluid pressure is equal to the hydrostatic pressure, plus the additional pressure that causes water to rise up in a well – the excess pressure that we discussed earlier. Therefore, we can combine equations 18-2 and 18-3 and rewrite the Terzaghi equation as equation 18-4:

\[ \sigma = \sigma' + \rho_w g z + u \]  

where:

- \( \sigma \) = total stress; also called the normal stress (if we are only considering vertical stresses)
- \( \sigma' \) = effective stress, or “grain to grain” stress in the rock
- \( \rho_w g z \) = hydrostatic pore fluid pressure
- \( u \) = excess fluid pressure

As this equation shows, the stresses are a function of depth, and the lack of exponents in the equations tells us that they are linearly proportional to depth. Therefore, we can draw a figure that graphically represents the distribution of each component of stress (lithostatic, hydrostatic, and excess fluid pressure) (Figure 18-6).

Figure 18-6A shows the relationship between stress and depth in an unconfined aquifer. Both the lithostatic and the pore fluid pressures increase linearly with depth; the pore fluid pressure is always a little less than half of the lithostatic. Figure 18-6B shows the relationship between stress and depth in a confined aquifer with an overlying water table. In the water table above the confining layer, the pore fluid pressure increases linearly; below the confining layer there is an additional component of pore fluid pressure (\( u \)). The additional pore fluid pressure decreases the effective stress accordingly – in other words, the pore fluid pressure pushes against the grains and reduces the stress at the grain contacts. The additional stress – \( u \) – represents the additional elevation of the total head above the water table.

Thought question: if the pore fluid pressure increases, it decreases the effective stress (in other words, it pushes against the stress of the weight of the overlying rock). What would happen if the fluid pressure increased so much that it equaled the lithostatic stress (i.e., if the effective stress were to go to zero)?
Compressibility and Storage Parameters

We have discussed stress; now let’s talk about strain and the relationship between strain and stress. In any given material, a certain amount of stress will result in a certain amount of strain. The ratio of strain to stress is called the compressibility. In terms more specific to what we are discussing here, the compressibility is the amount of shortening or extension (i.e., the change in thickness of a layer) for a change in pressure (i.e., fluid pressure). We can express this as equation 18-5:

\[ \alpha = \frac{\Delta L / L}{\Delta \sigma'} \]  

(Equation 18-5)

where:

- \( \alpha \) = the compressibility
- \( \Delta L / L \) = the change in length (thickness) over the total length
- \( \Delta \sigma' \) = the change in effective stress.

This is just a simple coefficient relating strain to stress, and it can be applied in a number of ways. If we know the compressibility of a material (e.g., a clay confining layer), we can calculate how much the layer will consolidate for a given change in head. To do that, we convert the change in head to a change in pressure, then multiply by the compressibility to calculate the change in length over the total length.

The specific storage is directly related to the compressibility of the aquifer by the following relationship (equation 18-6):

\[ S_s = \rho_w g (\alpha + \phi \beta) \]  

(Equation 18-6)

where:

- \( S_s \) = specific storage
- \( \rho_w \) = density of water
- \( g \) = gravity
- \( \alpha \) = the compressibility of the matrix
- \( \phi \) = the porosity
- \( \beta \) = the compressibility of water
Chapter 19 – Groundwater in Various Geologic Settings

Introduction

We’ve discussed various aspects of groundwater flow, ranging from the properties of porous media to the response of aquifers to a pumping well to the study of regional flow systems to the chemistry of groundwater systems. Most of our discussions dealt with idealized conceptual systems, like a perfectly confined aquifer, or a homogeneous layer of sand, or a set of evenly spaced fractures with a constant aperture. But as we all know, the real world is not like that. Groundwater occurs in a variety of different geologic settings, from unconsolidated alluvium to karstified limestone to fractured granite, and each geologic setting has specific geologic and hydrogeologic characteristics. To really study and understand hydrogeologic systems, you need to first understand the geology. In this chapter, we will look at the occurrence of groundwater in real geologic systems, and examine the relationship between geology and the water that is flowing through it.

Basic Classification of Hydrogeologic Environments

Groundwater occurs everywhere in the subsurface, and most any geologic setting can host an aquifer. However, there are certain geologic environments that tend to have sufficient porosity and permeability to support an aquifer. Keep in mind that both of these parameters are necessary – a clay or vuggy basalt unit may have lots of porosity and hold a lot of water, but if the pores are too small or not connected, permeability will be low and well yields will not be sufficient. Conversely, a set of fractures in a granite may have an extremely high permeability (remember, it only takes a small aperture to get a really permeable fracture), but if the matrix porosity is low, a well will quickly drain the fractures and will not yield much water in the long run.

To understand these systems, we will classify 5 basic geologic settings (with variations within each setting), plus a few special cases.

Alluvium

Alluvium refers to sediments deposited by running water. These include gravel in a stream channel, sand deposited on a river bank, and the mud that washes into people’s homes after a flood. Alluvial systems are often excellent aquifers. The natural sorting of sediments by flowing water often results in continuous deposits of coarse sediments that have high permeability and porosity. There are three basic types of alluvial deposits that we will consider here – alluvial fans, deltas, and floodplain/river valley deposits.

Alluvial Fans

Sediment transport in a stream is controlled by the energy of the stream – highly energetic streams in mountainous areas can carry lots of large grains, while slower streams with flatter stream gradients carry finer sediments. Whenever the gradient (and therefore, the energy) of a stream suddenly decreases, much of the sediments carried by the stream will suddenly drop out of the water and settle out. This occurs when a stream flows out of a mountainous area into a flat plain, leaving behind a big pile of sediment called an alluvial fan (Figures 19-1 and 19-2).

Figure 19-1. Cross section of an alluvial fan developing at the foot of a fault-block mountain range (© Uliana, 2001, 2012).
Streams flowing out onto alluvial fans tend to carry a wide variety of sediments, ranging from mud and silt to coarse gravel. The changes in stream energy across the fan result in sorting of sediments so that the coarser sediments are near the proximal end (i.e., near the mountain) and the finer sediments are near the distal end. Because of this, the most productive wells will generally be found up near the break in slope at the foot of the mountains. Sediments are generally unconsolidated, although in arid regions, extensive evaporation can cause mineral precipitation that partially cements the sediments. Also, evaporation can leave behind caliche – a layer of fairly hard calcium carbonate that inhibits infiltration of water into the fan.

Sometimes a series of alluvial fans will coalesce into a large package of sediments called a bajada, or alluvial apron (Figure 19-2). In fault-block mountain areas, like New Mexico and Nevada, entire valleys will be filled in with alluvium to depths of as much as 1000 meters.

Groundwater systems in alluvial fans are usually unconfined and receive recharge from infiltration of precipitation. Sometimes, interbedded sands and clays can result in locally-confined units. Groundwater flow is usually controlled by the topography; streams near the proximal end are usually losing flow to the aquifers, while the distal end of the fans often contain springs and gaining streams. These systems usually have very little fine material in them (as it gets carried away by streams), so permeabilities are often very high.

Alluvial-filled fault-block valleys often contain significant regional-scale groundwater flow systems (Figure 19-3).
In arid regions, most of the alluvium will be covered with a layer of caliche that inhibits infiltration of precipitation. Also, high evapotranspiration rates will cause most precipitation in the valleys to disappear before it gets to the water table. Recharge is usually through fractures in the bedrock exposed in the highlands; these fractures convey water to the unconsolidated alluvium. The flow system is towards the middle, with eventual discharge to shallow lakes in the center of the valley called *playa lakes*. In some places, groundwater discharges directly into the lake, and then flows out of the valley on the surface. In most arid and semi-arid regions, the lakes are usually dry and the primary mode of discharge is through evaporation directly from the water table.

Alluvial fans are often very important sources of water in arid/semi-arid regions because

- Much greater erosion and sedimentation in arid regions; fans are bigger and thicker
- Lack of surface water means that groundwater is more crucial.

Alluvial fans exist in humid regions, but are usually not as important.

**Deltas**

Deltas are like alluvial fans, but they form where a stream enters a larger body of water, such as a lake, inland sea, gulf, or ocean. As the stream enters the larger body of water, its velocity suddenly decreases drastically, and it drops almost all of its sediment. A big difference between alluvial fans and deltas is that deltas contain a lot more fine material – in alluvial fans, the water energy is so high that most of the fine material gets washed away, but in the typical delta, fluid energies are low, and coarse-grained sediments are only deposited in localized packets of sediment.

Deltas can be classified based on their overall shape and the forces that control their geometry. Deltas can be **fluvially-dominated**, **wave-dominated**, or **tidally-dominated**, or usually some combination of these (Figure 19-4).

![Fluvially-dominated delta](image)

Fluvially-dominated deltas (figure 19-4a) are deltas whose shape is controlled by the input of sediment. These form in relatively low-energy environments; usually some sort of inland sea or gulf. These are often called “birds-foot” deltas because the streams tend to form linear piles of sediment that extend out into the bay or ocean. The Mississippi delta is an example of a fluvially-dominated delta.

![Wave-dominated delta](image)

Wave-dominated deltas (figure 19-4b) form in areas with relatively high wave action. The shape of the delta is affected by the waves so that the delta front is sort of smoothed out. Wave action also results in a lot of re-working of the sediments in the delta front, which in turn results in removal of a lot of the fine sediments and sorting of the sand fractions. Several deltas on the Pacific Coast of North America have wave-dominated deltas.

![Tidally-dominated delta](image)

Tidally-dominated deltas (figure 19-4c) are similar to wave-dominated deltas in that there is extensive reworking of the sediments and the shape of the delta front. However, the energy associated with tides is usually perpendicular to the wave front, whereas the wave action creates a longshore drift that moves things parallel to the front. Tidally-dominated deltas tend to have lobes of sediment running perpendicular to the delta front.

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**Figure 19-4. Basic delta types (from Galloway and Hobday, 1996)**
In most deltas, the coarse-grained deposits are generally in abandoned channels. These channels are usually surrounded by mud and are isolated from each other. Therefore, the success of wells drilled into delta sediments depends upon hitting one of these channel deposits. In wave or tidally-dominated deltas, the delta fronts will be very sandy and will contain lots of water, but that water is usually non-potable sea water.

In general, deltas are usually not very good sources of potable water. Permeable units are too few and far between, and the water that is there is usually too salty to drink or contains too much organic material from swamps and tidal flat areas. Exceptions do occur; the Nile Delta in Egypt is generally very permeable, and water is clean enough to drink. However, high concentrations of manganese and iron causes their steel well screens to clog with rust and manganese oxides; usually their wells only last a few years before they have to drill a new one.

**Floodplains/River Valley Deposits**

The area around rivers can be very productive aquifers, and have historically been extensively used as a source of water for irrigation and drinking. These areas usually contain unconsolidated high-permeability coarse grained channel deposits interspersed with finer grained overbank deposits. They are also usually shallow and easy to drill, will interchange with surface streams and have a fairly constant source of recharge, and usually have fairly good water quality. In larger rivers, these deposits can be up to 50 meters thick and can have a significant saturated thickness.

In addition to current river valley deposits, floodplain and channel deposits from ancient rivers can often be found in glaciated areas or in deserts. In the Sahara Desert, for example, ground-penetrating radar has identified and extensive system of river valleys, buried beneath the sand, that contain vast groundwater reserves. Another example is the Ogallala aquifer, which is an extensive fluvial system that formed as the Rockies were uplifted in the middle of the country. This network of streams left behind extensive deposits of sands and silt that makes up one of the largest and most prolific aquifers in the world.

**Glacial Deposits**

During the Great Ice Age much of North America and Eurasia were covered by glaciers. After the ice retreated it left behind a cover of glacial sediment. Glacial sediments can also be found worldwide in high mountainous areas. Glacial deposits can be very permeable or very tight. We group all glacial deposits (called “drift”) into two classes - unstratified and stratified (or layered).

**Unstratified drift** is called “diamicton” or “till.” Diamicton is a general term describing any large deposit of geological material that contains clasts of many different sizes that are not well sorted. This could include certain mass wasting deposits (e.g., mudflows, avalanches, etc.). Till is a specific diamicton of known glacial origin. It consists of mixed-up sediments containing sediment grains ranging from boulders to very fine clays. The British use the very descriptive term “boulder-clay.” Till may be hard and compacted, although tills deposited upon or besides glaciers may grade into outwash. (Note: keep in mind that diamicton is a descriptive term, while till is an interpretive term)

Most well yields in till are very small. The poor sorting leads to low porosity and permeability. Productive wells must tap sandy layers or fractures in the till or else be located in a very sandy till.

**Stratified drift** is glacial deposits that are in layers. In contrast to till, these units can be very productive. Stratified glacial drift is generally formed by one of three processes:

- **Outwash deposits** have characteristics similar to alluvial systems because they are both formed by running water. Outwash can be found in vast “aprons” or outwash plains that spread out in front of the glacier. These are formed when melt water from the glacier runs out from beneath the ice and carries with it sediments from the glacier. Localized outwash deposits are found in “pitted” plains and as ice-contact stratified drift. These tend to be smaller deposits than outwash plains and may be more poorly sorted. Outwash can also be intercalated (“sandwiched”) between low permeability tills. Electrical resistivity is a useful exploration tool for finding the permeable layers in the low-permeability tills.

- **Productive river valley (floodplain) type deposits** are common in deglaciated areas. Some may be buried by glacial drift (0-150m thick); some form in valleys downstream from glacier termini. Buried
valleys can occur, and are frequently explored for with seismic, gravity, and electrical geophysical methods.

- **Eolian deposits** are sediments that were transported by wind. We identify two basic types of eolian deposits: dunes and loess. Sand dunes, if saturated, are excellent aquifers – they usually have well-sorted and relatively coarse sediment grains with little fine material. The widespread silty loess blankets are generally not good aquifers, as they are primarily silt and clay and have low porosities and permeabilities. However, they can supply domestic wells if they are fractured.

- **Lacustrine deposits** are also common in deglaciated areas. These form from small lakes that dry out, leaving behind deposits of very fine-grained material. The deposits end up being thinly layered, very low-permeability silts and clays that do not form productive aquifers. However, these can form non-continuous low permeability units that can support perched aquifers in overlying sandy units.

### Clastic Sedimentary Rocks – Sandstone/Shale

Similar to unconsolidated sediments, the porosity and permeability of sandstones is affected by sorting and grain size. Cementation also plays a big role. Well-cemented sandstones can have very low permeabilities, and have hydraulic properties that are more like granite than sandstone (e.g. Shawangunk Fm.). As a general rule, older sedimentary rocks (e.g. Silurian) are more cemented than younger (e.g. Miocene).

Other factors are involved in the suitability of sandstones as aquifers. The depositional system will determine the grain size, grain distribution, and overall geometry of the deposits. A beach or wave-dominated delta complex will usually result in continuous layers of well-sorted sand, while a fluvial or submarine channel system will tend to have the sandy parts in channel deposits that may not be well-connected. Some eolian deposits can cover vast areas (like the Nubian Sandstone in Africa) and support very productive aquifers. Cross-bedding, clay drapes, and other sedimentary structures can create anisotropies that may influence and control flow directions.

Structures related to tectonic and depositional stresses can also affect flow in sandstones. Growth faults associated with prograding delta sediments can create linear trends of high-sand deposits that are not well connected to each other. Folds and faults from tectonic stresses can create anisotropies and affect the overall geometry of the aquifer. Fractures can play a big role in transmitting precipitation from the surface to the aquifer, and can facilitate recharge. Fractures can greatly increase permeability in the subsurface; however, they can also fill in with post-diagenetic minerals and create low-permeability zones in the subsurface.

Shales are generally very poor aquifers, and usually act as confining zones when interbedded with sandstones. However, if the shales are fractured and are sufficiently thick, they can produce enough water to supply domestic water wells. In some places, like the Claire Valley of South Australia, the fractured shales are the most permeable units in the area and are the predominant source of water for the local vineyards.

### Chemical Sedimentary Rocks – Carbonates and Evaporites

Limestones, dolomites, and, to a lesser extent, evaporite like gypsum and halite, are widespread over the surface of the Earth. These rocks have an extremely wide range of hydrogeologic characteristics, and are responsible for some of the most productive aquifers in the world as well as extensive low permeability units with very low water quantity and quality.

Limestones can have a range of primary porosities and permeabilities. Limestones created from lime muds are usually very low, while ones created by the accumulation of invertebrate shells (e.g., coquinas) can have really high primary porosity. Since calcite (one of the primary carbonate minerals) is fairly soluble, these rocks can get well-cemented fairly easily, and limestones can have very little open pore space.

Limestones are also subject to extensive karstification, or dissolution. Karstification usually starts in fractures, and tends to occur at the water table where CO₂ can outgas from the system. As groundwater flows through fractures, small pores, and vugs, it dissolves the aragonite, calcite, and dolomite and enlarges the interconnected pores to create very high permeabilities. The resultant cavities may be filled with reddish silt, re-precipitated calcite, or they may remain open for fluid flow. Extensive karstification results in extremely transmissive and productive aquifers (e.g., the Edwards aquifer, the Floridian aquifer). These aquifers
respond quickly to recharge from storms, and flow rates are very high. They are also extremely susceptible to contamination.

Evaporite units very rarely form aquifers. They have low primary porosity, and are so ductile that fractures don’t stay open for long. Some aquifers in gypsum do occur (e.g., the Blaine aquifer of Texas and southwest Oklahoma), but these aquifers typically yield low-quality water and are limited in extent.

“Hard” Rocks

Granites, basalts, and metamorphic rocks are usually not very good aquifers. These rocks tend to have very low primary porosities, and in the ones that do have significant porosity (e.g., vuggy basalts), the porosity is often not well connected. There are exceptions to this; some granites will have a small amount of porosity along the grain boundaries that could be significant in a large enough thickness of the material. However, in general, productivity of these aquifers is almost always dependent on fractures.

These fractures can come from cooling stresses, from exfoliation and release of overburden stress, and from tectonic stresses. The key to studying and understanding these systems is by understanding the distribution of the fractures. In tectonically-dominated regions, the fractures will often follow fairly distinct orientations based on the structural history of the area. Cooling joints will tend to be vertical and fairly evenly spaced, although variations in spacing do occur.

Volcanic deposits can be stratified like sedimentary deposits, and the various layers will have different properties. Often we find basalt flows interbedded with ash-flow tuffs and other ash deposits. These will have different hydraulic properties, and will respond to fracturing differently (sometime cooling joints in the more brittle basalts will not propagate into the tuffs).
Chapter 20 – Water Law

Introduction

Water is a resource, just like minerals or petroleum, and there are laws that covers its development and use. The basic laws covering water use are administered at the state level, and individual states have different ways of managing water use. At the state level, surface water is frequently treated covered by different laws than groundwater. In this chapter, we will touch on the basics water law doctrines, first with surface water, then with groundwater.

Surface Water

The permission to use water from a stream is called a surface-water right. In the United States, surface water rights are determined by the individual state, and are handled in two basic ways.

Riparian doctrine

Under riparian doctrine, whoever owns land adjacent to a stream or other body of water has the right to withdrawal and use that water. In other words, all property owners adjacent to the body of water have an equal right to the water. Therefore, if an individual or community wishes to use water from a stream, they have to own land along that stream.

Everyone who owns land along a stream technically has an equal share in the water. One potential problem with this is that there is only so much water in a stream or lake, and often the law does not explicitly state how individuals are supposed to honor every other user's right to the water. There could be situations where one user is taking more than his share, and it is not easy to say what constitutes each person's fair share of the water.

This method of assigning water rights works best in areas where there tends to be plenty of water, and the issues of what quantity constitutes a fair share is not as significant. Most eastern states use Riparian doctrine to assign water rights.

Prior Appropriation

In western states, like Arizona, New Mexico, and Texas, there is much less rain and much more evaporation. Surface water is in much shorter supply, and it would be easy for any single landowner to pump more than their fair share from the rivers. Therefore, in most of these states, water law is based on the doctrine of Prior Appropriation. The basic idea behind Prior Appropriation is “first come-first served” – the right to withdrawal and use water is granted by the state to individuals, and priority for withdrawal is granted to the earliest individuals. For example:

- In 1940, the state grants a permit to Johnson City to withdrawal 1 million cubic meters of water per year. In 1963, the state grants the Town of Warfield a permit to withdrawal 500,000 cubic meters of water per year. The date that the permit is issued is called a priority date. It determines who has priority to withdrawal water. The user with the older priority date holds the senior permit; the one with the later date holds the junior permit. If it is a wet year and there is enough water to satisfy both permits, then each town can withdrawal their permitted amount. If there is a drought and there is not enough water to satisfy everyone, then Johnson City is allowed to withdrawal their full amount and the Town of Warfield can only take what is left.

Prior appropriation does not require land ownership - all a person needs is a permit from the state and some sort of access to the river or lake (e.g., an easement through private property for a pipeline). An advantage of prior appropriation is that the amounts that each user can withdrawal is clearly spelled out, and there is a mechanism for limiting how much people take during droughts. However, the question of how much water is available is a complicated one - discharge in a stream varies from place to place and from one day to the next.

A potential disadvantage is that water rights are maintained through continued use; if a user does not divert and use water for a period of time (e.g. seven years), then the water right is lost and the user has to apply for a new permit. This often leads to unnecessary use of water by users who do not want to lose their water right - a potential problem in water-poor areas.
Groundwater Rights

Groundwater law is more complicated than surface water law because most groundwater laws were developed before people really understood groundwater. There are two basic doctrines that govern groundwater withdrawal and use in the U.S.

**English Rule**

Under **English Rule**, also called the **Rule of Capture**, a landowner has an unlimited right to pump groundwater from the land they own (provided the water is not wasted or used irresponsibly). This is basically the same as Riparian doctrine - the landowner can take as much as they want. This doctrine came about when people thought that groundwater just sat under the ground waiting to be withdrawn. We now know that groundwater moves laterally, and pumping on one property can affect adjacent properties.

**American Rule**

**American Rule**, also called the **Rule of Reasonable Use**, limits water use by the use on the land above and contains provisions for limiting the impact of pumping on neighboring properties. The basic difference is that American Rule gives a landowner legal recourse to stop a neighbor from pumping so much that their water supply is threatened.

Some states have attempted to place limits on the amount of pumping by individuals, using a form of **appropriative rights** – in other words, the state grants permits for water use. California has addressed the groundwater rights problem by specifying that multiple landowners who have wells in the same aquifer are entitled to a share of the aquifer proportional to their share of the overlying land. This is called **correlative rights** (Figure 20-1).

![Figure 20-1. Correlative groundwater rights. Landowner A has a right to 50% of the water available in the aquifer; Landowner B has a right to 40%; etc. (© Uliana, 2001).](image-url)
References


